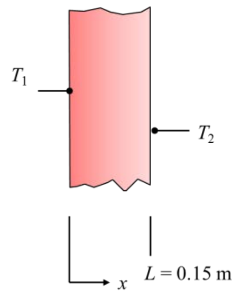
PROBLEM 1.1

KNOWN: Temperature distribution in wall of Example 1.1.

FIND: Heat fluxes and heat rates at $x = 0$ and $x = L$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through the wall, (2) constant thermal conductivity, (3) no internal thermal energy generation within the wall.

PROPERTIES: Thermal conductivity of wall (given): $k = 1.7$ W/m·K.

ANALYSIS: The heat flux in the wall is by conduction and is described by Fourier's law,

$$q''_x = -k\frac{dT}{dx} \tag{1}$$

Since the temperature distribution is $T(x) = a + bx$, the temperature gradient is

$$\frac{dT}{dx} = b \tag{2}$$

Hence, the heat flux is constant throughout the wall, and is

$$q''_x = -k\frac{dT}{dx} = -kb = -1.7\text{ W/m}\cdot\text{K}\times\left(-1000\text{ K/m}\right) = 1700\text{ W/m}^2 \quad <$$

Since the cross-sectional area through which heat is conducted is constant, the heat rate is constant and is

$$q_x = q''_x \times (W \times H) = 1700\text{ W/m}^2 \times \left(1.2\text{ m} \times 0.5\text{ m}\right) = 1020\text{ W} \quad <$$

Because the heat rate into the wall is equal to the heat rate out of the wall, steady-state conditions exist. $<$

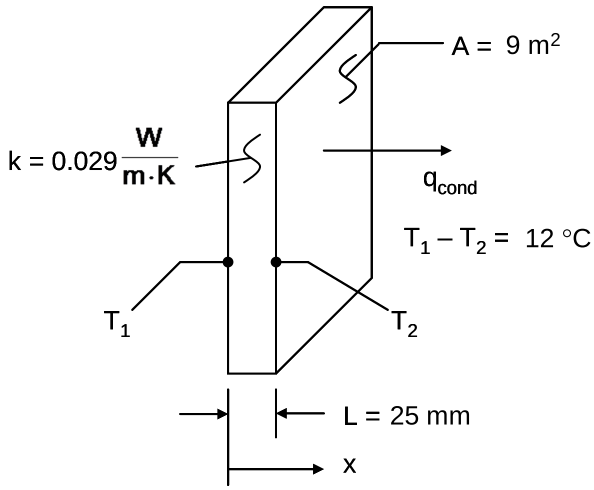
COMMENTS: (1) If the heat rates were not equal, the internal energy of the wall would be changing with time. (2) The temperatures of the wall surfaces are $T_1 = 1400$ K and $T_2 = 1250$ K.

PROBLEM 1.2

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation.

FIND: (a) The heat flux through a 3 m × 3 m sheet of the insulation, (b) the heat rate through the sheet, and (c) the thermal conduction resistance of the sheet.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: (a) From Equation 1.2 the heat flux is

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 0.029\frac{W}{m \cdot K} \times \frac{12\ K}{0.025\ m} = 13.9\frac{W}{m^2} \qquad <$$

(b) The heat rate is

$$q_x = q''_x \cdot A = 13.9\frac{W}{m^2} \times 9\ m^2 = 125\ W \qquad <$$

(c) From Eq. 1.11, the thermal resistance is

$$R_{t,cond} = \Delta T / q_x = 12\ K / 125\ W = 0.096\ K/W \qquad <$$

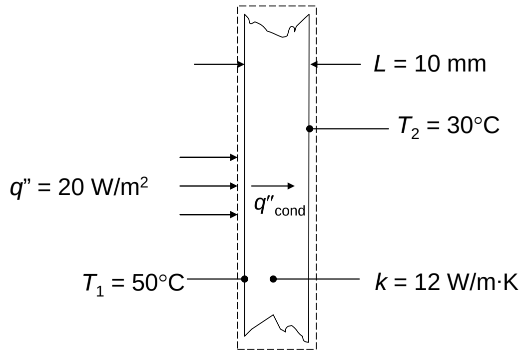
COMMENTS: (1) Be sure to keep in mind the important distinction between the heat flux (W/m^2) and the heat rate (W). (2) The direction of heat flow is from hot to cold. (3) Note that a temperature *difference* may be expressed in kelvins or degrees Celsius. (4) The conduction thermal resistance for a plane wall could equivalently be calculated from $R_{t,cond} = L/kA$.

PROBLEM 1.3

KNOWN: Thickness and thermal conductivity of a wall. Heat flux applied to one face and temperatures of both surfaces.

FIND: Whether steady-state conditions exist.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal energy generation.

ANALYSIS: Under steady-state conditions an energy balance on the control volume shown is

$$q''_{in} = q''_{out} = q''_{cond} = k(T_1 - T_2)/L = 12 \text{ W/m}\cdot\text{K}(50°\text{C} - 30°\text{C})/0.01 \text{ m} = 24{,}000 \text{ W/m}^2$$

Since the heat flux in at the left face is only 20 W/m^2, the conditions are not steady state. <

COMMENTS: If the same heat flux is maintained until steady-state conditions are reached, the steady-state temperature difference across the wall will be

$$\Delta T = q''L/k = 20 \text{ W/m}^2 \times 0.01 \text{ m}/12 \text{ W/m}\cdot\text{K} = 0.0167 \text{ K}$$

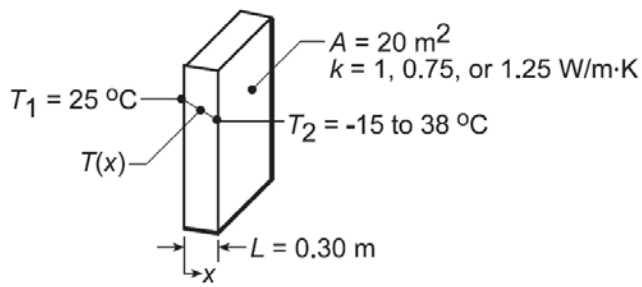
which is much smaller than the specified temperature difference of 20°C.

PROBLEM 1.4

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of outer surface temperatures ranging from -15 to 38°C.

SCHEMATIC:



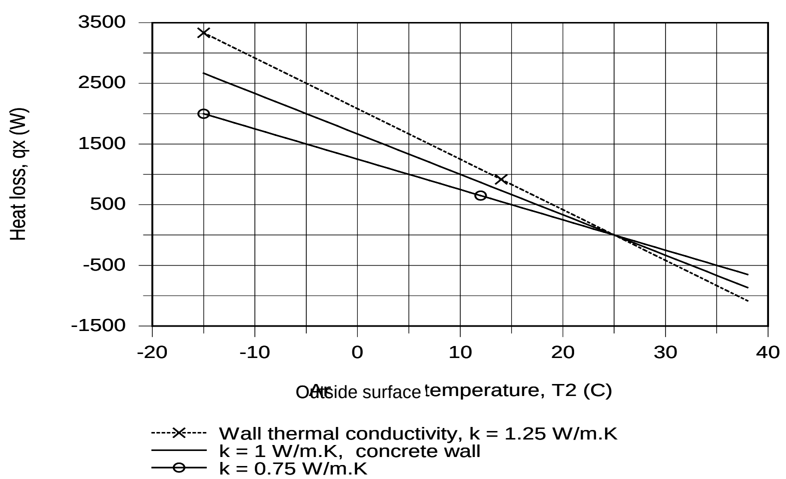
ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Fourier's law, if q''_x and k are each constant it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is T_2 = -15°C are

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,W/m \cdot K \frac{25^\circ C - \left(-15^\circ C\right)}{0.30\,m} = 133.3\,W/m^2 . \qquad (1)$$

$$q_x = q''_x \times A = 133.3\,W/m^2 \times 20\,m^2 = 2667\,W . \qquad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of outer surface temperature, $-15 \le T_2 \le 38$°C, with different wall thermal conductivities, k.



For the concrete wall, k = 1 W/m·K, the heat loss varies linearly from +2667 W to -867 W and is zero when the inside and outer surface temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

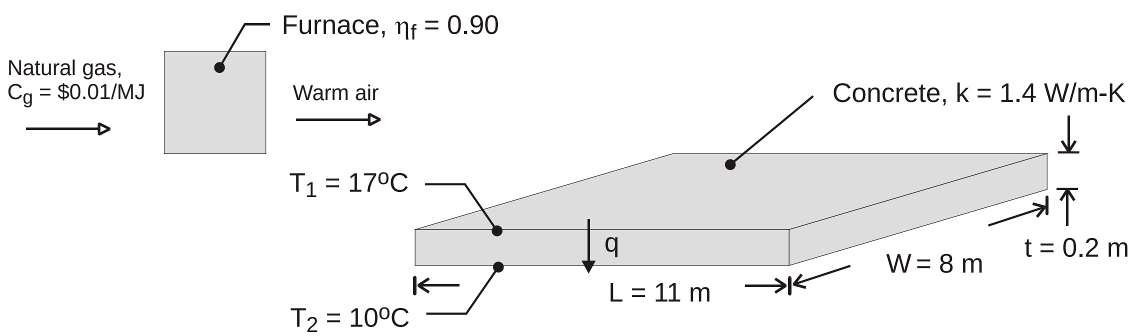
COMMENTS: Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

PROBLEM 1.5

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,\mathrm{W/m\cdot K}\,(11\,\mathrm{m}\times 8\,\mathrm{m})\frac{7^\circ\mathrm{C}}{0.20\,\mathrm{m}} = 4312\ \mathrm{W} \qquad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,\mathrm{W}\times \$0.02/\mathrm{MJ}}{0.9\times 10^6\,\mathrm{J/MJ}}(24\,\mathrm{h/d}\times 3600\,\mathrm{s/h}) = \$8.28/\mathrm{d} \qquad <$$

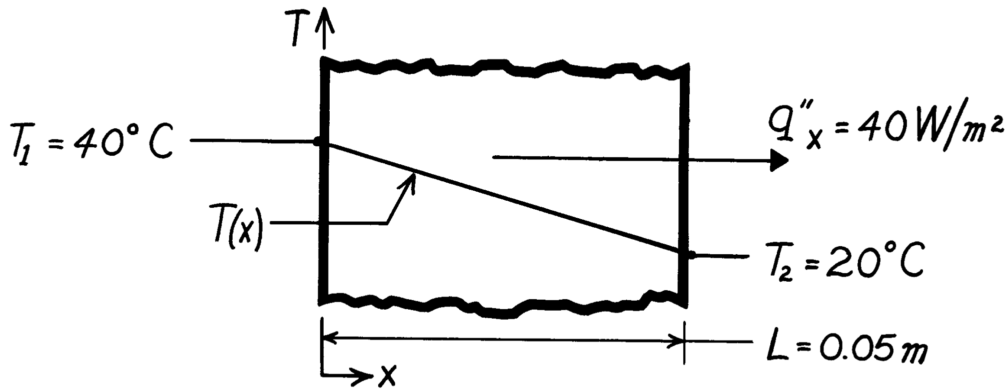
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.6

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^\circ C}$$

$$k = 0.10 \text{ W/m} \cdot \text{K}. \qquad <$$

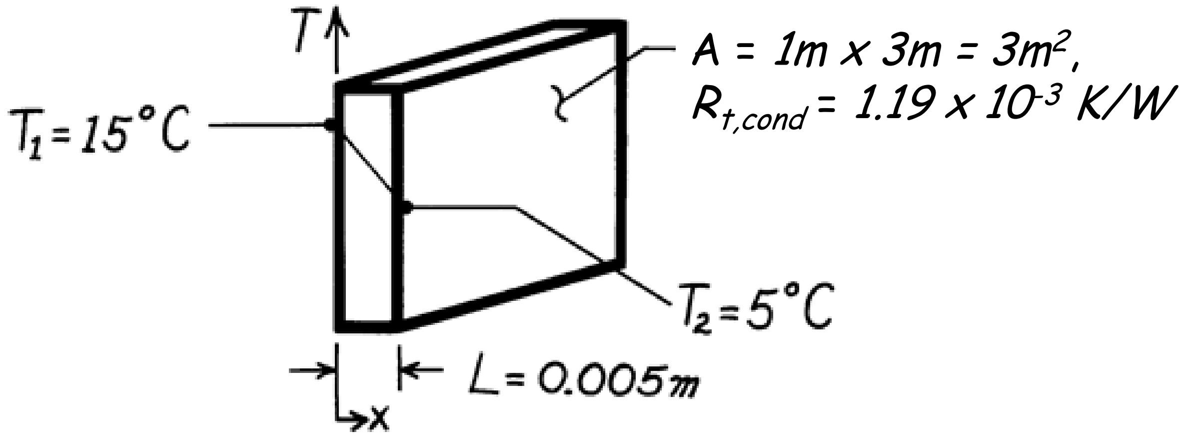
COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

PROBLEM 1.7

KNOWN: Inner and outer surface temperatures and thermal resistance of a glass window of prescribed dimensions.

FIND: Heat loss through window. Thermal conductivity of glass.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Eq. 1.11,

$$q_x = \frac{T_1 - T_2}{R_{t,cond}} = \frac{(15\text{-}5)^\circ \text{C}}{1.19 \times 10^{-3} \text{ K/W}} = 8400 \text{ W} \qquad <$$

The thermal resistance due to conduction for a plane wall is related to the thermal conductivity and dimensions according to

$$R_{t,cond} = L/kA$$

Therefore

$$k = L/(R_{t,cond}A) = 0.005 \text{ m} / (1.19 \times 10^{-3} \text{ K/W} \times 3 \text{ m}^2) = 1.40 \text{ W/m} \cdot \text{K} \qquad <$$

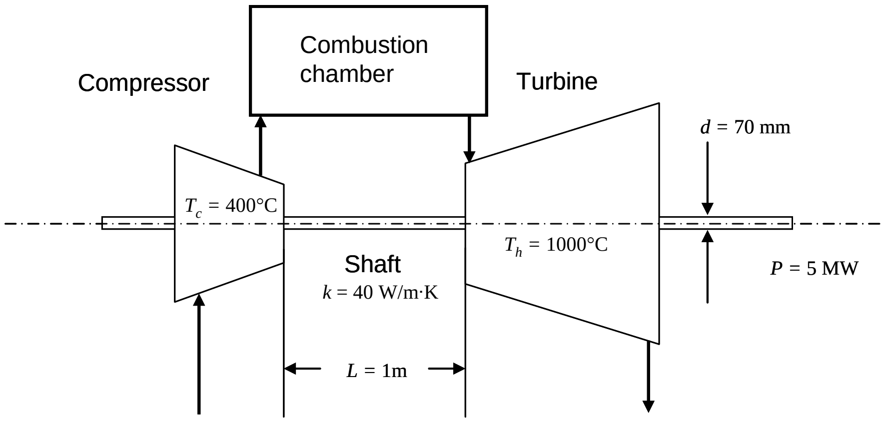
COMMENTS: The thermal conductivity value agrees with the value for glass in Table A.3.

PROBLEM 1.8

KNOWN: Net power output, average compressor and turbine temperatures, shaft dimensions and thermal conductivity.

FIND: (a) Comparison of the conduction rate through the shaft to the predicted net power output of the device, (b) Plot of the ratio of the shaft conduction heat rate to the anticipated net power output of the device over the range $0.005 \text{ m} \le L \le 1 \text{ m}$ and feasibility of a L = 0.005 m device.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Net power output is proportional to the volume of the gas turbine.

PROPERTIES: Shaft (given): k = 40 W/m·K.

ANALYSIS: (a) The conduction through the shaft may be evaluated using Fourier's law, yielding

$$q = q''A_c = \frac{k(T_h - T_c)}{L}\left(\pi d^2/4\right) = \frac{40\text{W/m}\cdot\text{K}(1000-400)^\circ\text{C}}{1\text{m}}\left(\pi(70\times10^{-3}\text{m})^2/4\right) = 92.4\text{W}$$

The ratio of the conduction heat rate to the net power output is

$$r = \frac{q}{P} = \frac{92.4\text{W}}{5\times10^6\text{W}} = 18.5\times10^{-6}$$ <

(b) The volume of the turbine is proportional to L^3. Designating L_a = 1 m, d_a = 70 mm and P_a as the shaft length, shaft diameter, and net power output, respectively, in part (a),

$$d = d_a \times (L/L_a);\ P = P_a \times (L/L_a)^3$$

and the ratio of the conduction heat rate to the net power output is

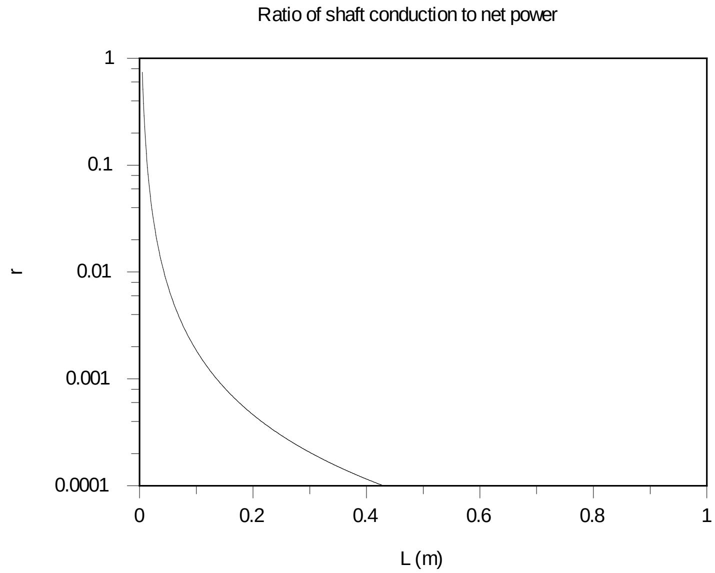
$$r = \frac{q''A_c}{P} = \frac{\frac{k(T_h - T_c)}{L}\left(\pi d^2/4\right)}{P} = \frac{\frac{k(T_h - T_c)}{L}\left(\pi\left(d_a L/L_a\right)^2/4\right)}{P_a(L/L_a)^3} = \frac{\frac{k(T_h - T_c)\pi}{4}d_a^2 L_a/P_a}{L^2}$$

$$= \frac{\frac{40\text{W/m}\cdot\text{K}(1000-400)^\circ\text{C}\pi}{4}(70\times10^{-3}\text{m})^2\times1\text{m}/5\times10^6\text{W}}{L^2} = \frac{18.5\times10^{-6}\text{m}^2}{L^2}$$

Continued…

PROBLEM 1.8 (Cont.)

The ratio of the shaft conduction to net power is shown below. At L = 0.005 m = 5 mm, the shaft conduction to net power output ratio is 0.74. The concept of the very small turbine is not feasible since it will be unlikely that the large temperature difference between the compressor and turbine can be maintained. <



COMMENTS: (1) The thermodynamics analysis does not account for heat transfer effects and is therefore meaningful only when heat transfer can be safely ignored, as is the case for the shaft in part (a). (2) Successful miniaturization of thermal devices is often hindered by heat transfer effects that must be overcome with innovative design.