Solutions Manual for DSP First
Second Edition

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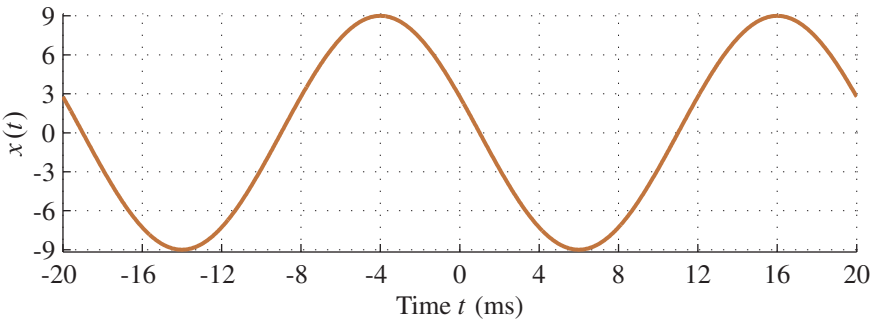
CHAPTER 2

Sinusoids

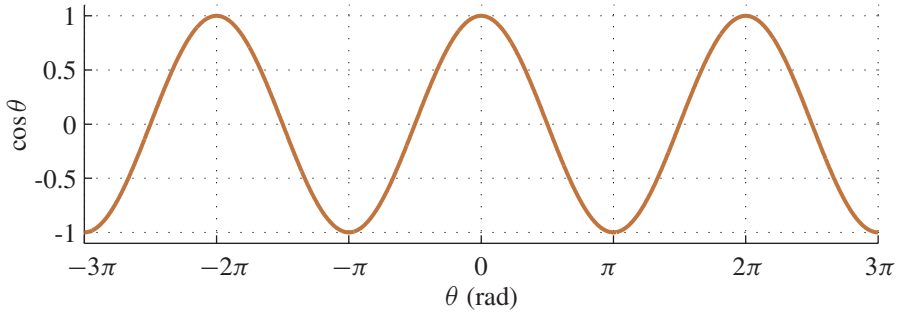
2-1 Problems



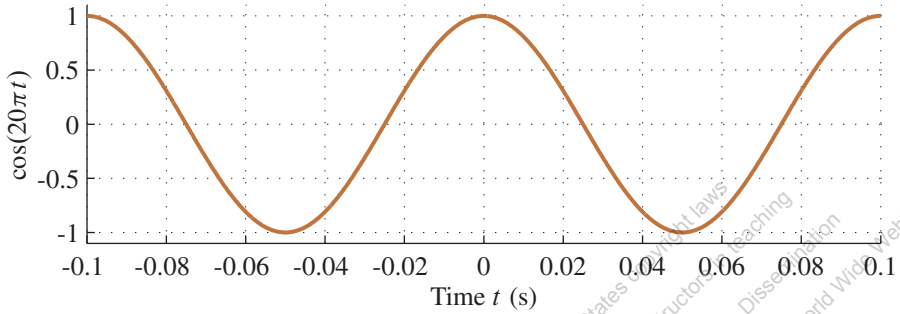
P-2.2 **DSP First 2e**

In the plot the period can be measured, $T = 12.5\,\text{ms} \Rightarrow \omega_0 = 2\pi/(12.5 \times 10^{-3}) = 2\pi(80)$ rad.
Positive peak closest to $t = 0$ is at $t_1 = 2.5\,\text{ms} \Rightarrow \varphi = -2\pi(2.5 \times 10^{-3})/(12.5 \times 10^{-3}) = 2\pi/5 = -0.4\pi$ rad.
Amplitude is $A = 8$.
$x(t) = 8\cos(160\pi t - 0.4\pi)$

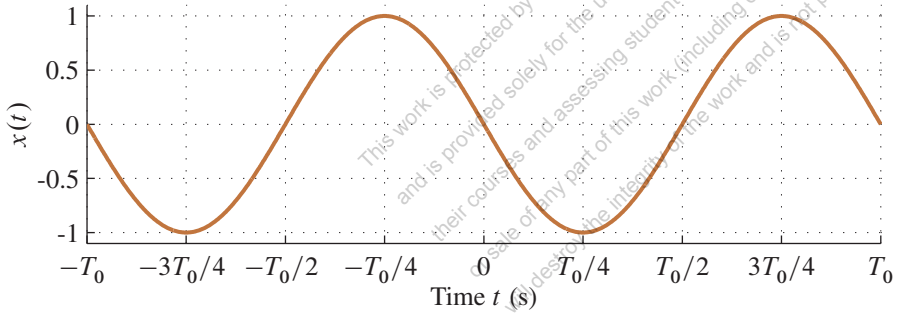
(a) Plot of $\cos\theta$



(b) Plot of $\cos(20\pi t)$



(c) Plot of $\cos(2\pi/T_0 + \pi/2)$



$$
\begin{aligned}
e^{j\theta} &= 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \cdots \\
&= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \cdots \\
&= \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right)}_{\cos\theta} + j\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\right)}_{\sin\theta}
\end{aligned}
$$

Thus, $e^{j\theta} = \cos\theta + j\sin\theta$

(a) Real part of complex exponential is cosine.

$$\begin{aligned}\cos(\theta_1+\theta_2) &= \Re\left\{e^{j(\theta_1+\theta_2)}\right\} = \Re\left\{e^{j\theta_1}e^{j\theta_2}\right\} \\ &= \Re\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)\} \\ &= \Re\{(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + j(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\}\end{aligned}$$

$$\cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

(b) Change the sign of θ_2.

$$\begin{aligned}\cos(\theta_1-\theta_2) &= \Re\left\{e^{j(\theta_1-\theta_2)}\right\} = \Re\left\{e^{j\theta_1}e^{-j\theta_2}\right\} \\ &= \Re\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)\} \\ &= \Re\{(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + j(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\}\end{aligned}$$

$$\cos(\theta_1-\theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$$

$$(\cos\theta + j\sin\theta)^n = \left(e^{j\theta}\right)^n = e^{jn\theta} = \cos(n\theta) + j\sin(n\theta)$$

$$\begin{aligned}\left(\tfrac{3}{5} + j\tfrac{4}{5}\right)^n &= \left(e^{j0.927}\right)^{100} = \left(e^{j0.295167\pi}\right)^{100} \\ &= e^{j29.5167\pi} \\ &= e^{j1.5167\pi}\, \cancel{e^{j28\pi}}^{\,1} \\ &= \cos(1.5167) + j\sin(1.5167) \\ &= 0.0525 - j0.9986\end{aligned}$$

(a) $3e^{j\pi/3} + 4e^{-j\pi/6} = 5e^{j0.12} = 4.9641 + j0.5981$

(b) $(\sqrt{3} - j3)^{10} = \left(\sqrt{12}e^{-j\pi/3}\right)^{10} = 248,832\underbrace{e^{-j10\pi/3}}_{e^{+j2\pi/3}} = -124,416 + j215,494.83$

(c) $(\sqrt{3} - j3)^{-1} = \left(\sqrt{12}e^{-j\pi/3}\right)^{-1} = (1/\sqrt{12})e^{+j\pi/3} = 0.2887e^{+j\pi/3} = 0.14434 + j0.25$

(d) $(\sqrt{3} - j3)^{1/3} = \left(\sqrt{12}e^{-j\pi/3}e^{j2\pi\ell}\right)^{1/3} = \left((12)^{1/6}e^{-j\pi/9}e^{j2\pi\ell/3}\right)$ for $\ell = 0, 1, 2$.
There are 3 answers:
$1.513e^{-j\pi/9} = 1.422 - j0.5175$
$1.513e^{-j7\pi/9} = -1.159 - j0.9726$
$1.513e^{-j13\pi/9} = 1.513e^{+j5\pi/9} = -0.2627 + j1.49$

(e) $\Re\left\{je^{-j\pi/3}\right\} = \Re\left\{e^{j\pi/2}e^{-j\pi/3}\right\} = \Re\left\{e^{j\pi/6}\right\} = \cos(\pi/6) = \sqrt{3}/2 = 0.866$