1 Circuit Variables

Assessment Problems

AP 1.1 Use a product of ratios to convert two-thirds the speed of light from meters per second to miles per second:

$$\left(\frac{2}{3}\right)\frac{3\times 10^8\ \text{m}}{1\ \text{s}}\cdot\frac{100\ \text{cm}}{1\ \text{m}}\cdot\frac{1\ \text{in}}{2.54\ \text{cm}}\cdot\frac{1\ \text{ft}}{12\ \text{in}}\cdot\frac{1\ \text{mile}}{5280\ \text{feet}}=\frac{124{,}274.24\ \text{miles}}{1\ \text{s}}.$$

Now set up a proportion to determine how long it takes this signal to travel 1100 miles:

$$\frac{124{,}274.24\ \text{miles}}{1\ \text{s}}=\frac{1100\ \text{miles}}{x\ \text{s}}.$$

Therefore,

$$x=\frac{1100}{124{,}274.24}=0.00885=8.85\times 10^{-3}\ \text{s}=8.85\ \text{ms}.$$

AP 1.2 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\$100\ \text{billion}\ =\$100\times 10^9.$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1\ \text{year}}{365.25\ \text{days}}\cdot\frac{1\ \text{day}}{24\ \text{hours}}\cdot\frac{1\ \text{hour}}{60\ \text{mins}}\cdot\frac{1\ \text{min}}{60\ \text{secs}}\cdot\frac{1\ \text{sec}}{1000\ \text{ms}}=\frac{1\ \text{year}}{31.5576\times 10^9\ \text{ms}}.$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100\times 10^9}{1\ \text{year}}\cdot\frac{1\ \text{year}}{31.5576\times 10^9\ \text{ms}}=\frac{100}{31.5576}=\$3.17/\text{ms}.$$

AP 1.3 Remember from Eq. 1.2, current is the time rate of change of charge, or $i = \frac{dq}{dt}$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. 1.2 to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x)\,dx.$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^\infty 20e^{-5000x}\,dx = \frac{20}{-5000}e^{-5000x}\Big|_0^\infty = \frac{20}{-5000}(e^{-\infty} - e^0)$$

$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004\text{ C} = 4000\,\mu\text{C}.$$

AP 1.4 Recall from Eq. 1.2 that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. 1.2:

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\right]$$

$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-\alpha t}\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= 0 - \left(\frac{1}{\alpha}e^{-\alpha t} - \alpha\frac{t}{\alpha}e^{-\alpha t}\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-\alpha t}$$

$$= te^{-\alpha t}.$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1 - \alpha t)e^{-\alpha t} = 0.$$

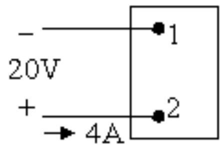
Since $e^{-\alpha t}$ never equals 0 for a finite value of t, the expression equals 0 only when $(1 - \alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-\alpha/\alpha} = \frac{1}{\alpha}e^{-1}.$$

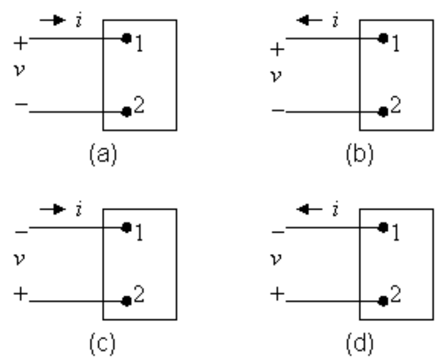
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10\text{ A}.$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



Also sketch the four figures from Fig. 1.6:



[a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

$$\text{(a) } v = -20\text{ V}, \quad i = -4\text{ A}; \quad \text{(b) } v = -20\text{ V}, \quad i = 4\text{ A};$$

$$\text{(c) } v = 20\text{ V}, \quad i = -4\text{ A}; \quad \text{(d) } v = 20\text{ V}, \quad i = 4\text{ A}.$$

[b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\text{ W}$.

[c] Since the power is greater than 0, the box is absorbing power.

AP 1.6 [a] Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. To find the time at which the power is maximum, find the first derivative of the power with respect to time, set the resulting expression equal to zero, and solve for time:

$$p = (80{,}000te^{-500t})(15te^{-500t}) = 120 \times 10^4 t^2 e^{-1000t};$$

$$\frac{dp}{dt} = 240 \times 10^4 te^{-1000t} - 120 \times 10^7 t^2 e^{-1000t} = 0.$$

Therefore,

$$240 \times 10^4 - 120 \times 10^7 t = 0.$$

Solving,

$$t = \frac{240 \times 10^4}{120 \times 10^7} = 2 \times 10^{-3} = 2 \text{ ms}.$$

[b] The maximum power occurs at 2 ms, so find the value of the power at 2 ms:

$$p(0.002) = 120 \times 10^4 (0.002)^2 e^{-2} = 649.6 \text{ mW}.$$

[c] From Eq. 1.3, we know that power is the time rate of change of energy, or $p = dw/dt$. If we know the power, we can find the energy by integrating Eq. 1.3. To find the total energy, the upper limit of the integral is infinity:

$$\begin{aligned} w_{\text{total}} &= \int_0^\infty 120 \times 10^4 x^2 e^{-1000x}\,dx \\ &= \frac{120 \times 10^4}{(-1000)^3} e^{-1000x}[(-1000)^2x^2 - 2(-1000)x + 2)\Big|_0^\infty \\ &= 0 - \frac{120 \times 10^4}{(-1000)^3} e^0(0 - 0 + 2) = 2.4 \text{ mJ}. \end{aligned}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}.$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 $(4\text{ cond.})\cdot(845\text{ mi})\cdot\frac{5280\text{ ft}}{1\text{ mi}}\cdot\frac{2526\text{ lb}}{1000\text{ ft}}\cdot\frac{1\text{ kg}}{2.2\text{ lb}}=20.5\times10^6\text{ kg}.$

P 1.2 **[a]** To begin, we calculate the number of pixels that make up the display:

$$n_{\text{pixels}}=(3840)(2160)=8{,}294{,}400\text{ pixels}.$$

Each pixel requires 24 bits of information. Since 8 bits equal one byte, each pixel requires 3 bytes of information. We can calculate the number of bytes of information required for the display by multiplying the number of pixels in the display by 3 bytes per pixel:

$$n_{\text{bytes}}=\frac{8{,}294{,}400\text{ pixels}}{1\text{ display}}\cdot\frac{3\text{ bytes}}{1\text{ pixel}}=24{,}883{,}200\text{ bytes/display}.$$

Finally, we use the fact that there are 10^6 bytes per MB:

$$\frac{24{,}883{,}200\text{ bytes}}{1\text{ display}}\cdot\frac{1\text{ MB}}{10^6\text{ bytes}}=24.88\text{ MB/display}.$$

[b] $$\frac{24{,}883{,}200\text{ bytes}}{1\text{ image}}\cdot\frac{30\text{ images}}{1\text{ s}}\cdot\frac{60\text{ s}}{1\text{ min}}\cdot\frac{60\text{ min}}{1\text{ hr}}\cdot\frac{2\text{ hr}}{1\text{ video}}$$

$$=5.375\times10^{12}\text{ bytes/video}=5.375\text{ TB/video}.$$

[c] $$\frac{24{,}883{,}200\text{ bytes}}{1\text{ image}}\cdot\frac{8\text{ bits}}{1\text{ byte}}\cdot\frac{30\text{ images}}{1\text{ sec}}=5{,}971{,}968{,}000\text{ bits/s}$$

$$=5.972\text{ Gb/s}.$$

P 1.3 **[a]** We can set up a ratio to determine how long it takes the bamboo to grow $10\,\mu$m First, recall that 1 mm $=10^3\mu$m. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250\text{ mm}}{1\text{ day}}\cdot\frac{1\text{ day}}{24\text{ hours}}\cdot\frac{1\text{ hour}}{60\text{ min}}\cdot\frac{1\text{ min}}{60\text{ sec}}=\frac{250}{(24)(60)(60)}=\frac{10}{3456}\text{ mm/s}.$$

Use a ratio to determine the time it takes for the bamboo to grow $10\,\mu$m:

$$\frac{10/3456\times10^{-3}\text{ m}}{1\text{ s}}=\frac{10\times10^{-6}\text{ m}}{x\text{ s}}\qquad\text{so}\qquad x=\frac{10\times10^{-6}}{10/3456\times10^{-3}}=3.456\text{ s}.$$

[b] $$\frac{1\text{ cell length}}{3.456\text{ s}}\cdot\frac{3600\text{ s}}{1\text{ hr}}\cdot\frac{(24)(7)\text{ hr}}{1\text{ week}}=175{,}000\text{ cell lengths/week}.$$

P 1.4 $$\frac{(480)(320)\text{ pixels}}{1\text{ frame}}\cdot\frac{2\text{ bytes}}{1\text{ pixel}}\cdot\frac{30\text{ frames}}{1\text{ sec}}=9.216\times10^6\text{ bytes/sec};$$

$$(9.216\times10^6\text{ bytes/sec})(x\text{ secs})=32\times2^{30}\text{ bytes};$$

$$x = \frac{32 \times 2^{30}}{9.216 \times 10^6} = 3728 \text{ sec} = 62 \text{ min} \approx 1 \text{ hour of video.}$$

P 1.5 [**a**] $\dfrac{20{,}000 \text{ photos}}{(11)(15)(1) \text{ mm}^3} = \dfrac{x \text{ photos}}{1 \text{ mm}^3};$

$$x = \frac{(20{,}000)(1)}{(11)(15)(1)} = 121 \text{ photos.}$$

[**b**] $\dfrac{16 \times 2^{30} \text{ bytes}}{(11)(15)(1) \text{ mm}^3} = \dfrac{x \text{ bytes}}{(0.2)^3 \text{ mm}^3};$

$$x = \frac{(16 \times 2^{30})(0.008)}{(11)(15)(1)} = 832{,}963 \text{ bytes.}$$

P 1.6 $\dfrac{(260 \times 10^6)(540)}{10^9} = 104.4$ gigawatt-hours.

P 1.7 First we use Eq. 1.2 to relate current and charge:

$$i = \frac{dq}{dt} = 24 \cos 4000t.$$

Therefore, $dq = 24 \cos 4000t\, dt$.

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 24 \int_0^t \cos 4000y\, dy.$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 24 \frac{\sin 4000y}{4000} \Big|_0^t = \frac{24}{4000} \sin 4000t - \frac{24}{4000} \sin 4000(0) = \frac{24}{4000} \sin 4000t.$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6 \times 10^{-3} \sin 4000t \text{ C} = 6 \sin 4000t \text{ mC}$.

P 1.8 $w = qV = (1.6022 \times 10^{-19})(6) = 9.61 \times 10^{-19} = 0.961$ aJ.

P 1.9 $n = \dfrac{35 \times 10^{-6} \text{ C/s}}{1.6022 \times 10^{-19} \text{ C/elec}} = 2.18 \times 10^{14}$ elec/s.

P 1.10 [**a**] First we use Eq. 1.2 to relate current and charge:

$$i = \frac{dq}{dt} = 0.125e^{-2500t}.$$

Therefore, $dq = 0.125e^{-2500t}\,dt$.

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 0.125\int_0^t e^{-2500y}\,dy.$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 0.125\frac{e^{-2500y}}{-2500}\bigg|_0^t = 50 \times 10^{-6}(1 - e^{-2500t}).$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 50(1 - e^{-2500t})\,\mu\text{C}.$$

[**b**] As $t \to \infty$, $q_T = 50\,\mu$C.

[**c**] $q(0.5 \times 10^{-3}) = (50 \times 10^{-6})(1 - e^{(-2500)(0.0005)}) = 35.675\,\mu$C.

P 1.11 [**a**] First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 40te^{-500t}.$$

Therefore, $dq = 40te^{-500t}\,dt$.

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 40\int_0^t ye^{-500y}\,dy.$$

We solve the integral and make the substitutions for the limits of the integral:

$$q(t) - q(0) = 40\frac{e^{-500y}}{(-500)^2}(-500y - 1)\bigg|_0^t = 160 \times 10^{-6}e^{-500t}(-500t - 1) + 160 \times 10^{-6}$$

$$= 160 \times 10^{-6}(1 - 500te^{-500t} - e^{-500t}).$$

But $q(0) = 0$ by hypothesis, so

$$q(t) = 160(1 - 500te^{-500t} - e^{-500t})\,\mu\text{C}.$$

[**b**] $q(0.001) = (160)[1 - 500(0.001)e^{-500(0.001)} - e^{-500(0.001)} = 14.4\,\mu$C.

P 1.12 [a] In Car B, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car B). Therefore using the passive sign convention,
$p = vi = (40)(12) = 480$ W.
Since the power is positive, the battery in Car B is absorbing power, so Car B must have the "dead" battery.

[b] $w(t) = \int_0^t p\,dx;\qquad 1.5\text{ min} = 1.5 \cdot \frac{60\text{ s}}{1\text{ min}} = 90\text{ s};$

$$w(90) = \int_0^{90} 480\,dx;$$

$$w = 480(90 - 0) = 480(90) = 43{,}200\text{ J} = 43.2\text{ kJ}.$$

P 1.13 Assume we are standing at box A looking toward box B. Use the passive sign convention to get $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (30)(6) = 180$ W 180 W from A to B;

[b] $p = (-20)(-8) = 160$ W 160 W from A to B;

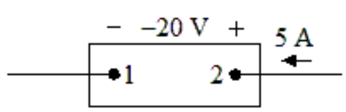
[c] $p = (-60)(4) = -240$ W 240 W from B to A;

[d] $p = (40)(-9) = -360$ W 360 W from B to A.

P 1.14 $p = (12)(0.1) = 1.2$ W; $\quad 4\text{ hr} \cdot \frac{3600\text{ s}}{1\text{ hr}} = 14{,}400\text{ s};$

$$w(t) = \int_0^t p\,dt;\qquad w(14{,}400) = \int_0^{14{,}400} 1.2\,dt = 1.2(14{,}400) = 17.28\text{ kJ}.$$

P 1.15 [a]



$p = vi = (-20)(5) = -100$ W.
Power is being delivered by the box.

[b] Entering.

[c] Gain.

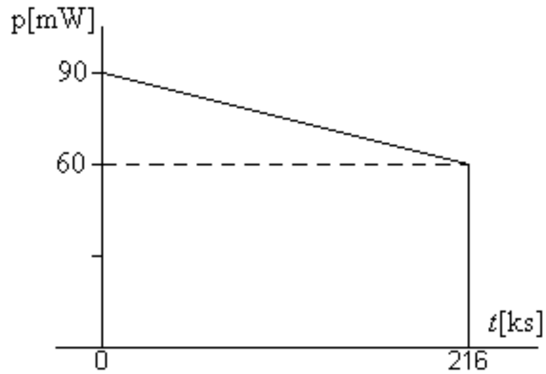
P 1.16 [a] $p = vi = (-20)(-5) = 100$ W, so power is being absorbed by the box.

[b] Leaving.

[c] Lose.

P 1.17 $p = vi; \qquad w = \int_0^t p\,dx.$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 60 hr = 216,000 s = 216 ks.

$$p(0) = (6)(15 \times 10^{-3}) = 90 \times 10^{-3} \text{ W};$$

$$p(216 \text{ ks}) = (4)(15 \times 10^{-3}) = 60 \times 10^{-3} \text{ W};$$

$$w = (60 \times 10^{-3})(216 \times 10^3) + \frac{1}{2}(90 \times 10^{-3} - 60 \times 10^{-3})(216 \times 10^3) = 16{,}200 \text{ J}.$$

P 1.18 **[a]** $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W;

$$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0 \qquad \text{so} \qquad 2e^{-2000t} = e^{-1000t};$$

$2 = e^{1000t}$ so $\ln 2 = 1000t$ thus p is maximum at $t = 693.15\,\mu$s;

$$p_{\text{max}} = p(693.15\,\mu\text{s}) = 937.5 \text{ mW}.$$

[b] $$w = \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\Big|_0^\infty\right]$$

$$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ}.$$

P 1.19 **[a]** $p = vi = (15e^{-250t})(0.04e^{-250t}) = 0.6e^{-500t}$ W;

$$p(0.01) = 0.6e^{-500(0.01)} = 0.6e^{-5} = 0.00404 = 4.04 \text{ mW}.$$

[b] $$w_{\text{total}} = \int_0^\infty p(x)\,dx = \int_0^\infty 0.6e^{-500x}\,dx = \frac{0.6}{-500}e^{-500x}\Big|_0^\infty$$

$$= -0.0012(e^{-\infty} - e^0) = 0.0012 = 1.2 \text{ mJ}.$$

P 1.20 **[a]**

$$\begin{aligned} p &= vi \\ &= [(1500t+1)e^{-750t}](0.04e^{-750t}) \\ &= (60t+0.04)e^{-1500t}; \\ \frac{dp}{dt} &= 60e^{-1500t} - 1500e^{-1500t}(60t+0.04) \\ &= -90{,}000te^{-1500t}. \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so $p_{\max}$ occurs at $t = 0$.

[b]

$$\begin{aligned} p_{\max} &= [(60)(0) + 0.04]e^0 = 0.04 \\ &= 40 \text{ mW}. \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p dx \\ &= \int_0^t 60xe^{-1500x}\, dx + \int_0^t 0.04e^{-1500x}\, dx \\ &= \frac{60e^{-1500x}}{(-1500)^2}(-1500x-1)\bigg|_0^t + 0.04\frac{e^{-1500x}}{-1500}\bigg|_0^t. \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{60}{225 \times 10^4} + \frac{0.04}{1500} = 53.33\,\mu\text{J}.$$

P 1.21 **[a]** $p = vi = 0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}$;
$p(625\,\mu\text{s}) = 42.2$ mW.

[b]

$$\begin{aligned} w(t) &= \int_0^t (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}) \\ &= 140.625 - 78.125e^{-3200t} + 250e^{-2000t} - 312.5e^{-800t}\mu\text{J}; \\ w(625\,\mu\text{s}) &= 12.14\,\mu\text{J}. \end{aligned}$$

[c] $w_{\text{total}} = 140.625\,\mu\text{J}$.

P 1.22 **[a]**

$$\begin{aligned} p &= vi = [10^4t+5)e^{-400t}][(40t+0.05)e^{-400t}] \\ &= 400 \times 10^3t^2e^{-800t} + 700te^{-800t} + 0.25e^{-800t} \\ &= e^{-800t}[400{,}000t^2 + 700t + 0.25]; \\ \frac{dp}{dt} &= \{e^{-800t}[800 \times 10^3t + 700] - 800e^{-800t}[400{,}000t^2 + 700t + 0.25]\} \\ &= [-3{,}200{,}000t^2 + 2400t + 5]100e^{-800t}. \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $3{,}200{,}000t^2 - 2400t - 5 = 0$
so $p_{\max}$ occurs at $t = 1.68$ ms.

[b]

$$\begin{aligned} p_{\max} &= [400{,}000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)} \\ &= 666.34 \text{ mW}. \end{aligned}$$