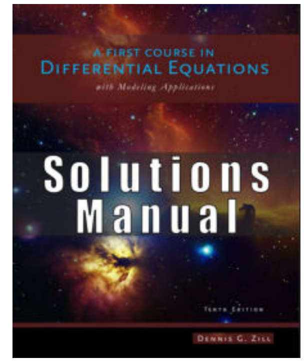


Chapter 1

Introduction to Differential Equations

1.1 Definitions and Terminology

1. Second order; linear

2. Third order; nonlinear because of $(dy/dx)^4$

3. Fourth order; linear

4. Second order; nonlinear because of $\cos(r+u)$

5. Second order; nonlinear because of $(dy/dx)^2$ or $\sqrt{1+(dy/dx)^2}$

6. Second order; nonlinear because of R^2

7. Third order; linear

8. Second order; nonlinear because of $\dot{x}^2$

9. Writing the differential equation in the form $x(dy/dx)+y^2=1$, we see that it is nonlinear in y because of y^2. However, writing it in the form $(y^2-1)(dx/dy)+x=0$, we see that it is linear in x.

10. Writing the differential equation in the form $u(dv/du)+(1+u)v=ue^u$ we see that it is linear in v. However, writing it in the form $(v+uv-ue^u)(du/dv)+u=0$, we see that it is nonlinear in u.

11. From $y=e^{-x/2}$ we obtain $y'=-\frac{1}{2}e^{-x/2}$. Then $2y'+y=-e^{-x/2}+e^{-x/2}=0$.

12. From $y=\frac{6}{5}-\frac{6}{5}e^{-20t}$ we obtain $dy/dt=24e^{-20t}$, so that

$$\frac{dy}{dt}+20y=24e^{-20t}+20\left(\frac{6}{5}-\frac{6}{5}e^{-20t}\right)=24.$$

13. From $y=e^{3x}\cos 2x$ we obtain $y'=3e^{3x}\cos 2x-2e^{3x}\sin 2x$ and $y''=5e^{3x}\cos 2x-12e^{3x}\sin 2x$, so that $y''-6y'+13y=0$.

14. From $y = -\cos x \ln(\sec x + \tan x)$ we obtain $y' = -1 + \sin x \ln(\sec x + \tan x)$ and $y'' = \tan x + \cos x \ln(\sec x + \tan x)$. Then $y'' + y = \tan x$.

15. The domain of the function, found by solving $x+2 \geq 0$, is $[-2, \infty)$. From $y' = 1+2(x+2)^{-1/2}$ we have

$$\begin{aligned}(y-x)y' &= (y-x)[1 + (2(x+2)^{-1/2}]\\ &= y - x + 2(y-x)(x+2)^{-1/2}\\ &= y - x + 2[x + 4(x+2)^{1/2} - x](x+2)^{-1/2}\\ &= y - x + 8(x+2)^{1/2}(x+2)^{-1/2} = y - x + 8.\end{aligned}$$

An interval of definition for the solution of the differential equation is $(-2, \infty)$ because y' is not defined at $x = -2$.

16. Since $\tan x$ is not defined for $x = \pi/2 + n\pi$, n an integer, the domain of $y = 5\tan 5x$ is $\{x \mid 5x \neq \pi/2 + n\pi\}$
or $\{x \mid x \neq \pi/10 + n\pi/5\}$. From $y' = 25\sec^2 5x$ we have

$$y' = 25(1 + \tan^2 5x) = 25 + 25\tan^2 5x = 25 + y^2.$$

An interval of definition for the solution of the differential equation is $(-\pi/10, \pi/10)$. Another interval is $(\pi/10, 3\pi/10)$, and so on.

17. The domain of the function is $\{x \mid 4 - x^2 \neq 0\}$ or $\{x \mid x \neq -2 \text{ and } x \neq 2\}$. From $y' = 2x/(4 - x^2)^2$ we have

$$y' = 2x\left(\frac{1}{4 - x^2}\right)^2 = 2xy^2.$$

An interval of definition for the solution of the differential equation is $(-2, 2)$. Other intervals are $(-\infty, -2)$ and $(2, \infty)$.

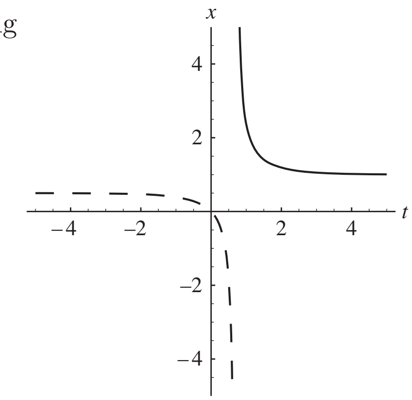
18. The function is $y = 1/\sqrt{1 - \sin x}$, whose domain is obtained from $1 - \sin x \neq 0$ or $\sin x \neq 1$. Thus, the domain is $\{x \mid x \neq \pi/2 + 2n\pi\}$. From $y' = -\frac{1}{2}(1 - \sin x)^{-3/2}(-\cos x)$ we have

$$2y' = (1 - \sin x)^{-3/2}\cos x = [(1 - \sin x)^{-1/2}]^3 \cos x = y^3 \cos x.$$

An interval of definition for the solution of the differential equation is $(\pi/2, 5\pi/2)$. Another one is $(5\pi/2, 9\pi/2)$, and so on.

19. Writing $\ln(2X-1)-\ln(X-1)=t$ and differentiating implicitly we obtain

$$\begin{aligned}\frac{2}{2X-1}\frac{dX}{dt}-\frac{1}{X-1}\frac{dX}{dt}&=1\\ \left(\frac{2}{2X-1}-\frac{1}{X-1}\right)\frac{dX}{dt}&=1\\ \frac{2X-2-2X+1}{(2X-1)(X-1)}\frac{dX}{dt}&=1\\ \frac{dX}{dt}=-(2X-1)(X-1)&=(X-1)(1-2X).\end{aligned}$$



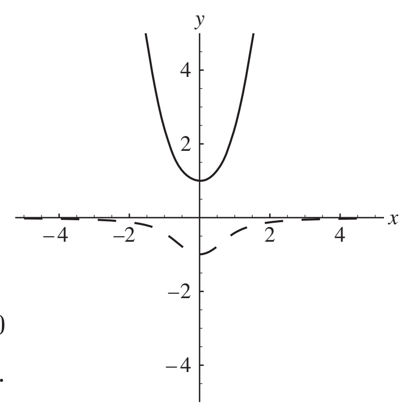
Exponentiating both sides of the implicit solution we obtain

$$\begin{aligned}\frac{2X-1}{X-1}&=e^t\\ 2X-1&=Xe^t-e^t\\ (e^t-1)&=(e^t-2)X\\ X&=\frac{e^t-1}{e^t-2}.\end{aligned}$$

Solving $e^t-2=0$ we get $t=\ln 2$. Thus, the solution is defined on $(-\infty,\ln 2)$ or on $(\ln 2,\infty)$. The graph of the solution defined on $(-\infty,\ln 2)$ is dashed, and the graph of the solution defined on $(\ln 2,\infty)$ is solid.

20. Implicitly differentiating the solution, we obtain

$$\begin{aligned}-2x^2\frac{dy}{dx}-4xy+2y\frac{dy}{dx}&=0\\ -x^2\,dy-2xy\,dx+y\,dy&=0\\ 2xy\,dx+(x^2-y)dy&=0.\end{aligned}$$



Using the quadratic formula to solve $y^2-2x^2y-1=0$ for y, we get $y=\left(2x^2\pm\sqrt{4x^4+4}\right)/2=x^2\pm\sqrt{x^4+1}$. Thus, two explicit solutions are $y_1=x^2+\sqrt{x^4+1}$ and $y_2=x^2-\sqrt{x^4+1}$. Both solutions are defined on $(-\infty,\infty)$. The graph of $y_1(x)$ is solid and the graph of y_2 is dashed.

21. Differentiating $P = c_1 e^t / \left(1 + c_1 e^t\right)$ we obtain

$$\frac{dP}{dt} = \frac{\left(1 + c_1 e^t\right) c_1 e^t - c_1 e^t \cdot c_1 e^t}{\left(1 + c_1 e^t\right)^2} = \frac{c_1 e^t}{1 + c_1 e^t} \frac{\left[\left(1 + c_1 e^t\right) - c_1 e^t\right]}{1 + c_1 e^t}$$

$$= \frac{c_1 e^t}{1 + c_1 e^t} \left[1 - \frac{c_1 e^t}{1 + c_1 e^t}\right] = P(1 - P).$$

22. Differentiating $y = 2x^2 - 1 + c_1 e^{-2x^2}$ we obtain $\dfrac{dy}{dx} = 4x - 4xc_1 e^{-2x^2}$, so that

$$\frac{dy}{dx} + 4xy = 4x - 4xc_1 e^{-2x^2} + 8x^3 - 4x + 4c_1 x e^{-x^2} = 8x^3$$

23. From $y = c_1 e^{2x} + c_2 x e^{2x}$ we obtain $\dfrac{dy}{dx} = (2c_1 + c_2)e^{2x} + 2c_2 x e^{2x}$ and $\dfrac{d^2y}{dx^2} = (4c_1 + 4c_2)e^{2x} + 4c_2 x e^{2x}$, so that

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = (4c_1 + 4c_2 - 8c_1 - 4c_2 + 4c_1)e^{2x} + (4c_2 - 8c_2 + 4c_2)xe^{2x} = 0.$$

24. From $y = c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2$ we obtain

$$\frac{dy}{dx} = -c_1 x^{-2} + c_2 + c_3 + c_3 \ln x + 8x,$$

$$\frac{d^2y}{dx^2} = 2c_1 x^{-3} + c_3 x^{-1} + 8,$$

and

$$\frac{d^3y}{dx^3} = -6c_1 x^{-4} - c_3 x^{-2},$$

so that

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (-6c_1 + 4c_1 + c_1 + c_1)x^{-1} + (-c_3 + 2c_3 - c_2 - c_3 + c_2)x$$
$$+ (-c_3 + c_3)x \ln x + (16 - 8 + 4)x^2$$
$$= 12x^2$$

In Problems 25–28, we use the Product Rule and the derivative of an integral ((12) of this section): $\dfrac{d}{dx}\displaystyle\int_a^x g(t)\,dt = g(x)$.

25. Differentiating $y = e^{3x} \displaystyle\int_1^x \frac{e^{-3t}}{t}\,dt$ we obtain $\dfrac{dy}{dx} = 3e^{3x} \displaystyle\int_1^x \frac{e^{-3t}}{t}\,dt + \frac{e^{-3x}}{x} \cdot e^{3x}$ or $\dfrac{dy}{dx} = 3e^{3x} \displaystyle\int_1^x \frac{e^{-3t}}{t}\,dt + \frac{1}{x}$, so that

$$x\frac{dy}{dx} - 3xy = x\left(3e^{3x} \int_1^x \frac{e^{-3t}}{t}\,dt + \frac{1}{x}\right) - 3x\left(e^{3x} \int_1^x \frac{e^{-3t}}{t}\,dt\right)$$

$$= 3xe^{3x} \int_1^x \frac{e^{-3t}}{t}\,dt + 1 - 3xe^{3x} \int_1^x \frac{e^{-3t}}{t}\,dt = 1$$

26. Differentiating $y = \sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt$ we obtain $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt + \frac{\cos x}{\sqrt{x}}\cdot\sqrt{x}$ or $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt + \cos x$, so that

$$2x\frac{dy}{dx} - y = 2x\left(\frac{1}{2\sqrt{x}}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt + \cos x\right) - \sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt$$

$$= \sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt + 2x\cos x - \sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt = 2x\cos x$$

27. Differentiating $y = \frac{5}{x} + \frac{10}{x}\int_1^x \frac{\sin t}{t}\,dt$ we obtain $\frac{dy}{dx} = -\frac{5}{x^2} - \frac{10}{x^2}\int_1^x \frac{\sin t}{t}\,dt + \frac{\sin x}{x}\cdot\frac{10}{x}$ or $\frac{dy}{dx} = -\frac{5}{x^2} - \frac{10}{x^2}\int_1^x \frac{\sin t}{t}\,dt + \frac{10\sin x}{x^2}$, so that

$$x^2\frac{dy}{dx} + xy = x^2\left(-\frac{5}{x^2} - \frac{10}{x^2}\int_1^x \frac{\sin t}{t}\,dt + \frac{10\sin x}{x^2}\right) + x\left(\frac{5}{x} + \frac{10}{x}\int_1^x \frac{\sin t}{t}\,dt\right)$$

$$= -5 - 10\int_1^x \frac{\sin t}{t}\,dt + 10\sin x + 5 + 10\int_1^x \frac{\sin t}{t}\,dt = 10\sin x$$

28. Differentiating $y = e^{-x^2} + e^{-x^2}\int_0^x e^{t^2}\,dt$ we obtain $\frac{dy}{dx} = -2xe^{-x^2} - 2xe^{-x^2}\int_0^x e^{t^2}\,dt + e^{x^2}\cdot e^{-x^2}$ or $\frac{dy}{dx} = -2xe^{-x^2} - 2xe^{-x^2}\int_0^x e^{t^2}\,dt + 1$, so that

$$\frac{dy}{dx} + 2xy = \left(-2xe^{-x^2} - 2xe^{-x^2}\int_0^x e^{t^2}\,dt + 1\right) + 2x\left(e^{-x^2} + e^{-x^2}\int_0^x e^{t^2}\,dt\right)$$

$$= -2xe^{-x^2} - 2xe^{-x^2}\int_0^x e^{t^2}\,dt + 1 + 2xe^{-x^2} + 2xe^{-x^2}\int_0^x e^{t^2}\,dt = 1$$

29. From

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \ge 0 \end{cases}$$

we obtain

$$y' = \begin{cases} -2x, & x < 0 \\ 2x, & x \ge 0 \end{cases}$$

so that $xy' - 2y = 0$.

30. The function $y(x)$ is not continuous at $x = 0$ since $\lim_{x\to 0^-} y(x) = 5$ and $\lim_{x\to 0^+} y(x) = -5$. Thus, $y'(x)$ does not exist at $x = 0$.

31. Substitute the function $y = e^{mx}$ into the equation $y' + 2y = 0$ to get

$$(e^{mx})' + 2(e^{mx}) = 0$$

$$me^{mx} + 2e^{mx} = 0$$

$$e^{mx}(m + 2) = 0$$

Now since $e^{mx} > 0$ for all values of x, we must have $m = -2$ and so $y = e^{-2x}$ is a solution.

32. Substitute the function $y = e^{mx}$ into the equation $5y' - 2y = 0$ to get

$$5(e^{mx})' - 2(e^{mx}) = 0$$
$$5me^{mx} - 2e^{mx} = 0$$
$$e^{mx}(5m - 2) = 0$$

Now since $e^{mx} > 0$ for all values of x, we must have $m = 2/5$ and so $y = e^{2x/5}$ is a solution.

33. Substitute the function $y = e^{mx}$ into the equation $y'' - 5y' + 6y = 0$ to get

$$(e^{mx})'' - 5(e^{mx})' + 6(e^{mx}) = 0$$
$$m^2e^{mx} - 5me^{mx} + 6e^{mx} = 0$$
$$e^{mx}(m^2 - 5m + 6) = 0$$
$$e^{mx}(m - 2)(m - 3) = 0$$

Now since $e^{mx} > 0$ for all values of x, we must have $m = 2$ or $m = 3$ therefore $y = e^{2x}$ and $y = e^{3x}$ are solutions.

34. Substitute the function $y = e^{mx}$ into the equation $2y'' + 7y' - 4y = 0$ to get

$$2(e^{mx})'' + 7(e^{mx})' - 4(e^{mx}) = 0$$
$$2m^2e^{mx} + 7me^{mx} - 4e^{mx} = 0$$
$$e^{mx}(2m^2 + 7m - 4) = 0$$
$$e^{mx}(m + 4)(2m - 1) = 0$$

Now since $e^{mx} > 0$ for all values of x , we must have $m = -4$ or $m = 1/2$ therefore $y = e^{-4x}$ and $y = e^{x/2}$ are solutions.

35. Substitute the function $y = x^m$ into the equation $xy'' + 2y' = 0$ to get

$$x \cdot (x^m)'' + 2(x^m)' = 0$$
$$x \cdot m(m - 1)x^{m-2} + 2mx^{m-1} = 0$$
$$(m^2 - m)x^{m-1} + 2mx^{m-1} = 0$$
$$x^{m-1}[m^2 + m] = 0$$
$$x^{m-1}[m(m + 1)] = 0$$

The last line implies that $m = 0$ or $m = -1$ therefore $y = x^0 = 1$ and $y = x^{-1}$ are solutions.

36. Substitute the function $y = x^m$ into the equation $x^2y'' - 7xy' + 15y = 0$ to get

$$\begin{aligned} x^2 \cdot (x^m)'' - 7x \cdot (x^m)' + 15(x^m) &= 0 \\ x^2 \cdot m(m-1)x^{m-2} - 7x \cdot mx^{m-1} + 15x^m &= 0 \\ (m^2 - m)x^m - 7mx^m + 15x^m &= 0 \\ x^m[m^2 - 8m + 15] &= 0 \\ x^m[(m-3)(m-5)] &= 0 \end{aligned}$$

The last line implies that $m = 3$ or $m = 5$ therefore $y = x^3$ and $y = x^5$ are solutions.

In Problems 37–40, we substitute $y = c$ into the differential equations and use $y' = 0$ and $y'' = 0$

37. Solving $5c = 10$ we see that $y = 2$ is a constant solution.

38. Solving $c^2 + 2c - 3 = (c+3)(c-1) = 0$ we see that $y = -3$ and $y = 1$ are constant solutions.

39. Since $1/(c-1) = 0$ has no solutions, the differential equation has no constant solutions.

40. Solving $6c = 10$ we see that $y = 5/3$ is a constant solution.

41. From $x = e^{-2t} + 3e^{6t}$ and $y = -e^{-2t} + 5e^{6t}$ we obtain

$$\frac{dx}{dt} = -2e^{-2t} + 18e^{6t} \quad \text{and} \quad \frac{dy}{dt} = 2e^{-2t} + 30e^{6t}.$$

Then

$$x + 3y = (e^{-2t} + 3e^{6t}) + 3(-e^{-2t} + 5e^{6t}) = -2e^{-2t} + 18e^{6t} = \frac{dx}{dt}$$

and

$$5x + 3y = 5(e^{-2t} + 3e^{6t}) + 3(-e^{-2t} + 5e^{6t}) = 2e^{-2t} + 30e^{6t} = \frac{dy}{dt}.$$

42. From $x = \cos 2t + \sin 2t + \frac{1}{5}e^t$ and $y = -\cos 2t - \sin 2t - \frac{1}{5}e^t$ we obtain

$$\frac{dx}{dt} = -2\sin 2t + 2\cos 2t + \frac{1}{5}e^t \quad \text{and} \quad \frac{dy}{dt} = 2\sin 2t - 2\cos 2t - \frac{1}{5}e^t$$

and

$$\frac{d^2x}{dt^2} = -4\cos 2t - 4\sin 2t + \frac{1}{5}e^t \quad \text{and} \quad \frac{d^2y}{dt^2} = 4\cos 2t + 4\sin 2t - \frac{1}{5}e^t.$$

Then

$$4y + e^t = 4(-\cos 2t - \sin 2t - \frac{1}{5}e^t) + e^t = -4\cos 2t - 4\sin 2t + \frac{1}{5}e^t = \frac{d^2x}{dt^2}$$

and

$$4x - e^t = 4(\cos 2t + \sin 2t + \frac{1}{5}e^t) - e^t = 4\cos 2t + 4\sin 2t - \frac{1}{5}e^t = \frac{d^2y}{dt^2}.$$

43. $(y')^2 + 1 = 0$ has no real solutions because $(y')^2 + 1$ is positive for all differentiable functions $y = \phi(x)$.

44. The only solution of $(y')^2 + y^2 = 0$ is $y = 0$, since if $y \neq 0$, $y^2 > 0$ and $(y')^2 + y^2 \geq y^2 > 0$.

45. The first derivative of $f(x) = e^x$ is e^x. The first derivative of $f(x) = e^{kx}$ is ke^{kx}. The differential equations are $y' = y$ and $y' = ky$, respectively.

46. Any function of the form $y = ce^x$ or $y = ce^{-x}$ is its own second derivative. The corresponding differential equation is $y'' - y = 0$. Functions of the form $y = c\sin x$ or $y = c\cos x$ have second derivatives that are the negatives of themselves. The differential equation is $y'' + y = 0$.

47. We first note that $\sqrt{1-y^2} = \sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} = |\cos x|$. This prompts us to consider values of x for which $\cos x < 0$, such as $x = \pi$. In this case

$$\left.\frac{dy}{dx}\right|_{x=\pi} = \left.\frac{d}{dx}(\sin x)\right|_{x=\pi} = \cos x\big|_{x=\pi} = \cos\pi = -1,$$

but

$$\sqrt{1-y^2}|_{x=\pi} = \sqrt{1-\sin^2\pi} = \sqrt{1} = 1.$$

Thus, $y = \sin x$ will only be a solution of $y' = \sqrt{1-y^2}$ when $\cos x > 0$. An interval of definition is then $(-\pi/2, \pi/2)$. Other intervals are $(3\pi/2, 5\pi/2)$, $(7\pi/2, 9\pi/2)$, and so on.

48. Since the first and second derivatives of $\sin t$ and $\cos t$ involve $\sin t$ and $\cos t$, it is plausible that a linear combination of these functions, $A\sin t + B\cos t$, could be a solution of the differential equation. Using $y' = A\cos t - B\sin t$ and $y'' = -A\sin t - B\cos t$ and substituting into the differential equation we get

$$\begin{aligned} y'' + 2y' + 4y &= -A\sin t - B\cos t + 2A\cos t - 2B\sin t + 4A\sin t + 4B\cos t \\ &= (3A - 2B)\sin t + (2A + 3B)\cos t = 5\sin t \end{aligned}$$

Thus $3A - 2B = 5$ and $2A + 3B = 0$. Solving these simultaneous equations we find $A = \frac{15}{13}$ and $B = -\frac{10}{13}$. A particular solution is $y = \frac{15}{13}\sin t - \frac{10}{13}\cos t$.

49. One solution is given by the upper portion of the graph with domain approximately $(0, 2.6)$. The other solution is given by the lower portion of the graph, also with domain approximately $(0, 2.6)$.

50. One solution, with domain approximately $(-\infty, 1.6)$ is the portion of the graph in the second quadrant together with the lower part of the graph in the first quadrant. A second solution, with domain approximately $(0, 1.6)$ is the upper part of the graph in the first quadrant. The third solution, with domain $(0, \infty)$, is the part of the graph in the fourth quadrant.

51. Differentiating $(x^3+y^3)/xy = 3c$ we obtain

$$\frac{xy(3x^2+3y^2y') - (x^3+y^3)(xy'+y)}{x^2y^2} = 0$$

$$3x^3y + 3xy^3y' - x^4y' - x^3y - xy^3y' - y^4 = 0$$

$$(3xy^3 - x^4 - xy^3)y' = -3x^3y + x^3y + y^4$$

$$y' = \frac{y^4 - 2x^3y}{2xy^3 - x^4} = \frac{y(y^3-2x^3)}{x(2y^3-x^3)}.$$

52. A tangent line will be vertical where y' is undefined, or in this case, where $x(2y^3 - x^3) = 0$. This gives $x = 0$ or $2y^3 = x^3$. Substituting $y^3 = x^3/2$ into $x^3 + y^3 = 3xy$ we get

$$x^3 + \frac{1}{2}x^3 = 3x\left(\frac{1}{2^{1/3}}\,x\right)$$

$$\frac{3}{2}x^3 = \frac{3}{2^{1/3}}x^2$$

$$x^3 = 2^{2/3}x^2$$

$$x^2(x - 2^{2/3}) = 0.$$

Thus, there are vertical tangent lines at $x = 0$ and $x = 2^{2/3}$, or at $(0,0)$ and $(2^{2/3}, 2^{1/3})$. Since $2^{2/3} \approx 1.59$, the estimates of the domains in Problem 50 were close.

53. The derivatives of the functions are $\phi_1'(x) = -x/\sqrt{25-x^2}$ and $\phi_2'(x) = x/\sqrt{25-x^2}$, neither of which is defined at $x = \pm 5$.

54. To determine if a solution curve passes through $(0,3)$ we let $t = 0$ and $P = 3$ in the equation $P = c_1e^t/(1+c_1e^t)$. This gives $3 = c_1/(1+c_1)$ or $c_1 = -\frac{3}{2}$. Thus, the solution curve

$$P = \frac{(-3/2)e^t}{1-(3/2)e^t} = \frac{-3e^t}{2-3e^t}$$

passes through the point $(0,3)$. Similarly, letting $t = 0$ and $P = 1$ in the equation for the one-parameter family of solutions gives $1 = c_1/(1+c_1)$ or $c_1 = 1 + c_1$. Since this equation has no solution, no solution curve passes through $(0,1)$.

55. For the first-order differential equation integrate $f(x)$. For the second-order differential equation integrate twice. In the latter case we get $y = \int(\int f(x)dx)\,dx + c_1x + c_2$.

56. Solving for y' using the quadratic formula we obtain the two differential equations

$$y' = \frac{1}{x}\left(2 + 2\sqrt{1+3x^6}\right) \quad \text{and} \quad y' = \frac{1}{x}\left(2 - 2\sqrt{1+3x^6}\right),$$

so the differential equation cannot be put in the form $dy/dx = f(x,y)$.