CHAPTER 1: ALGEBRA AND ARITHMETIC REVIEWS AND SELF-TESTS

In this chapter we present a review of basic arithmetic and algebra along with a series of tests designed to help you identify your initial strengths and weaknesses. You should read this chapter during the first few days of your course. It is possible that the instructor will not be covering this basic material in class.

The chapter is in two sections. Section 1.1 reviews basic arithmetic and section 1.2 reviews basic algebra. Each section is in turn composed of three parts: a pretest, a review, and an exit test. Use the pretest to identify your strengths. If you have difficulties, you should read the review part carefully. If you have no difficulties, you could just refresh your memory by reading the review quickly. The exit test helps you assess what you have learned from studying the review. If you remain unsure, you should discuss your problems with the instructor. Answers to all self-tests are on page 26.

1.1 Review of Basic Arithmetic

Pretest

1. What is 1/10 of 3/4?

a) 1/10

a) 15/5

b) 2/15

b) 3/40

c) None of the above

2. One number is 3 more than 2 times another. Their sum is 21. Find the numbers.

a) 7, 14

a) 2, 19

b) 6, 15

c) 10, 11

d) 8, 13

3. Evaluate c1-fig-5001.jpg.

a) c1-fig-5002.jpg

b) c1-fig-5003.jpg

c) c1-fig-5004.jpg

d) c1-fig-5005.jpg

e) None of the above

4. What is the product of c1-fig-5006.jpg?

a) c1-fig-5007.jpg

b) −12

c) c1-fig-5008.jpg

d) c1-fig-5009.jpg

e) 9

5. What is the value of the expression 1/[1+1/(1+1/4)]?

a) 9=5

b) 5=9

c) 1=2

d) 3

e) 5

6. Which of the following has the smallest value?

a) 1/0.2

b) 0.1/2

c) 0.1/1

d) 0.2/0.1

e) 2/0.2

7. Which is the smallest number?

a) $5(10^{-5})/3(10^{-5})$

b) 0.3/0.2

c) $0.3/0.3(10^{-4})$

d) $5(10^{-3})/0.1$

e) $0.3/0.3(10^{-2})$

8. Evaluate $10^3 + 10^5$.

a) 10^8

b) 10^{15}

c) 20^8

d) 2^{10}

e) 101,000

9. Evaluate $-10 - \{(2^3 + 27)/[3 - 2(8 - 10)]\}$.

a) 5

b) −15

c) 25

d) 10

e) 35

10. Which is the largest fraction?

a) 1/5

b) 2/9

c) 2/11
d) 4/16
e) 3/19

11. Evaluate c1-fig-5010.jpg.

a) 4
b) −4
c) 16
d) 1/8
e) 1/4

12. Evaluate $(2^{100} + 2^{98})/(2^{100} - 2^{98})$.

a) 2^{198}
b) 2^{99}
c) 64
d) 8
e) 5/3

13. Evaluate $(2^{-4} + 2^{-1})/2^{-3}$.

a) 9/27
b) 18
c) 1/2
d) 2^{-3}
e) 9/2

14. What is 1/10 of 3/8?

a) 1/8
b) 15/4
c) 15/2
d) 4/15
e) None of the above

15. Evaluate 3/6 + 2/12.

a) 1/12

b) 5/6

c) 2/3

d) 8/9

e) 5/18

16. Simplify c1-fig-5011.jpg.

a) c1-fig-5012.jpg

b) c1-fig-5013.jpg

c) c1-fig-5014.jpg

d) c1-fig-5015.jpg

e) 50

17. Evaluate c1-fig-5016.jpg.

a) c1-fig-5017.jpg

b) c1-fig-5018.jpg

c) 0

d) 3

e) c1-fig-5019.jpg

18. The cheapest among the following prices is

a) 10 oz for 16 cents

b) 2 oz for 3 cents

c) 4 oz for 7 cents

d) 20 oz for 34 cents

e) 8 oz for 13 cents

19. Evaluate $|4 + (-3)| + |-2|$.

a) -2

b) -1

c) 1
d) 3
e) 9

20. Evaluate $|-42||7|$.

a) -294
b) -49
c) -35
d) 284
e) 294

ARITHMETIC REVIEW

Introduction to the Real Number System

Arithmetic is concerned with certain operations like addition, subtraction, multiplication, and division carried out on numbers and the relations between numbers expressed by such phrases as “greater than” or “less than.”

We will illustrate how the general concept of a real number can serve as the basis of a mathematical theory. Let us suppose that we wish to measure the interval AB by means of the interval CD being the unit of measurement in figure 1.1

c1-fig-0001.jpg

Figure 1.1

We apply the interval CD to AB by determining how many times CD fits into AB. Suppose that this occurs n_0 times. If, after doing this, there is a remainder PB, then we divide the interval CD into ten parts and measure the remainder with these tenths. Suppose that n_1 of the tenths go to the remainder. If after this there is still a remainder, we divide our new measure (the tenth of CD) into ten parts again (i.e., we divide CD into a hundred parts) and repeat the same operation. Either the process of measurement comes to an end or it continues. In either case we reach the result that in the interval AB the whole interval CD is contained n_0 times, the tenths are contained n_1 times, the hundredths are contained n_2 times, and so on. In other words, we derive the ratio AB to CD with increasing accuracy: up to tenths, to hundredths, and so on. The ratio itself is represented by a decimal fraction with n_0 units, n_1 tenths, n_2 hundredths, and so on, or

$$(AB/CD) = n_0 n_1 n_2 n_3 \dots$$

The decimal fraction may be infinite, corresponding to the possibility of an indefinite increase in the precision of measurement.

The ratio of two intervals or of any two magnitudes in general is always representable by a decimal fraction, either finite or infinite. A real number may be formally defined as a finite or infinite decimal fraction.

Our definition will be complete if we say what we mean by the operations of addition and multiplication for decimal fractions. This is done in such a way that the operations defined on decimal fractions correspond to the operations on the magnitudes themselves. Therefore, when intervals are put together, their lengths are added; that is, the length of the interval AB + BC equals the sum of the lengths AB and BC. In defining the operations on real numbers, the difficulty is that these numbers are represented in general by infinite decimal fractions, while the well-known rules for these operations refer to finite decimal fractions. A rigorous definition of the operations for infinite decimals may be made in the following way. Suppose, for example, that we must add the two numbers a and b. We take the corresponding decimal fractions up to a given decimal place, say the millionth, and add them up. We thus obtain the sum $a + b$ with corresponding accuracy up to two millionths, since the errors in “a” and “b” may be added together. We are able to define the sum of two numbers with an arbitrary degree of accuracy and in that sense their sum is completely defined up to the chosen degree of accuracy.

We will define the following collections of numbers on which we will define arithmetic and algebraic operations.

The collection of **natural numbers** is denoted by

$$Z_+ = \{1, 2, 3, \dots\}$$

and contains **all positive integers**. Adding zero to the above collection yields the set of **whole numbers** denoted by

$$Z_0 = \{0, 1, 2, 3, \ldots\}$$

whereas adding to the above all negative integers yields the collection of **all integers** denoted by

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

The next important collection of numbers is that of **rational** numbers.

Definition 1.1: A rational number is a number that can be expressed as the ratio of two integers, the divisor, of course, not being zero. The set or collection of all rational numbers is denoted by Q.

For example 3/5, 5, and 1/2 are all rational numbers, since they are expressed as the ratio a/b where "a" and "b" are integers. In this case for $3/5 = a/b$, $a = 3$, and $b = 5$. For $5 = a/b$, $a = 5$, and $b = 1$, and for $1/2 = a/b$, we have that $a = 1$ and $b = 2$. Rational numbers can be expressed in decimal form. In the above examples $1/2 = 0.5$ and $3/5 = 0.6$.

Every rational number can be expressed as a **terminating** or a **periodic** decimal. By a periodic decimal we mean a number that includes a repeating portion in its decimal part. For instance,

$$\begin{aligned} 1/3 &= 0.333\dot{3} \\ 1/6 &= 0.166\dot{6} \\ 1/11 &= 0.0909\dot{0}\dot{9} \end{aligned}$$

are periodic decimals. In a sense all integers can be thought of as periodic decimal numbers with period 0. In that case

$$4 = 4.0000$$

and so on.

Any number that cannot be expressed as a terminating or periodic decimal is **irrational**. An irrational number therefore is a nonperiodic decimal. Examples of irrational numbers are

$$\sqrt{2} = 1.414221356\ldots$$
$$\pi = 3.141519265\ldots$$

and

$$0.101001000100001\ \ldots$$

The last number has a pattern that is not periodic. The set of all irrational numbers is denoted by $\bar{Q}$.

Squaring a number means multiplying the number by itself. In this case

$$2^2 = 2 \times 2 = 4$$

$$3^2 = 3 \times 3 = 9$$

Finding the **square root** is the inverse operation to squaring a number. Numbers such as c1-fig-5023.jpg, c1-fig-5024.jpg, c1-fig-5025.jpg and c1-fig-5026.jpg are called **radicals.** Note that there are two square roots to any number, a positive and a negative one. For instance, the square roots of 4 are 2 and −2, since $2^2 = (-2)^2 = 4$. Some square roots cannot be expressed by a periodic decimal such as

c1-fig-5027.jpg

These are all irrational numbers.

Real numbers

The set of real numbers c1-fig-5028.jpg is the set of periodic and non-periodic decimals. We can also define c1-fig-5028.jpg as the union of the set of rational numbers Q and the set of irrational numbers $\bar{Q}$.

Examples of real numbers are 1, −2, 3, c1-fig-5023.jpg, 2/3, 1/3, c1-fig-5025.jpg etc, where c1-fig-5023.jpg and c1-fig-5025.jpg are irrational. Each real number can be associated with