1

UNITS, PHYSICAL QUANTITIES, AND VECTORS

1.1. **IDENTIFY:** Convert units from mi to km and from km to ft.

SET UP: $1 \text{ in.} = 2.54 \text{ cm}$, $1 \text{ km} = 1000 \text{ m}$, $12 \text{ in.} = 1 \text{ ft}$, $1 \text{ mi} = 5280 \text{ ft}$.

EXECUTE: **(a)** $1.00 \text{ mi} = (1.00 \text{ mi})\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)\left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) = 1.61 \text{ km}$

(b) $1.00 \text{ km} = (1.00 \text{ km})\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{10^2 \text{ cm}}{1 \text{ m}}\right)\left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 3.28 \times 10^3 \text{ ft}$

EVALUATE: A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

1.2. **IDENTIFY:** Convert volume units from L to in.^3.

SET UP: $1 \text{ L} = 1000 \text{ cm}^3$. $1 \text{ in.} = 2.54 \text{ cm}$

EXECUTE: $0.473 \text{ L} \times \left(\frac{1000 \text{ cm}^3}{1 \text{ L}}\right) \times \left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right)^3 = 28.9 \text{ in.}^3$.

EVALUATE: 1 in.^3 is greater than 1 cm^3, so the volume in in.^3 is a smaller number than the volume in cm^3, which is 473 cm^3.

1.3. **IDENTIFY:** We know the speed of light in m/s. $t = d/v$. Convert 1.00 ft to m and t from s to ns.

SET UP: The speed of light is $v = 3.00 \times 10^8$ m/s. 1 ft = 0.3048 m. $1 \text{ s} = 10^9$ ns.

EXECUTE: $t = \frac{0.3048 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.02 \times 10^{-9} \text{ s} = 1.02 \text{ ns}$

EVALUATE: In 1.00 s light travels $3.00 \times 10^8 \text{ m} = 3.00 \times 10^5 \text{ km} = 1.86 \times 10^5 \text{ mi}$.

1.4. **IDENTIFY:** Convert the units from g to kg and from cm^3 to m^3.

SET UP: $1 \text{ kg} = 1000 \text{ g}$. $1 \text{ m} = 100 \text{ cm}$.

EXECUTE: $19.3 \frac{\text{g}}{\text{cm}^3} \times \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 1.93 \times 10^4 \frac{\text{kg}}{\text{m}^3}$

EVALUATE: The ratio that converts cm to m is cubed, because we need to convert cm^3 to m^3.

1.5. **IDENTIFY:** Convert volume units from in.^3 to L.

SET UP: $1 \text{ L} = 1000 \text{ cm}^3$. $1 \text{ in.} = 2.54 \text{ cm}$.

EXECUTE: $(327 \text{ in.}^3) \times (2.54 \text{ cm/in.})^3 \times (1 \text{ L}/1000 \text{ cm}^3) = 5.36 \text{ L}$

EVALUATE: The volume is 5360 cm^3. 1 cm^3 is less than 1 in.^3, so the volume in cm^3 is a larger number than the volume in in.^3.

1.6. **IDENTIFY:** Convert ft^2 to m^2 and then to hectares.

SET UP: $1.00 \text{ hectare} = 1.00\times10^4 \text{ m}^2$. $1 \text{ ft} = 0.3048 \text{ m}$.

EXECUTE: The area is $(12.0 \text{ acres})\left(\dfrac{43{,}600 \text{ ft}^2}{1 \text{ acre}}\right)\left(\dfrac{0.3048 \text{ m}}{1.00 \text{ ft}}\right)^2\left(\dfrac{1.00 \text{ hectare}}{1.00\times10^4 \text{ m}^2}\right) = 4.86 \text{ hectares}.$

EVALUATE: Since $1 \text{ ft} = 0.3048 \text{ m}$, $1 \text{ ft}^2 = (0.3048)^2 \text{ m}^2$.

1.7. **IDENTIFY:** Convert seconds to years. 1 gigasecond is a billion seconds.

SET UP: $1 \text{ gigasecond} = 1\times10^9 \text{ s}$. $1 \text{ day} = 24 \text{ h}$. $1 \text{ h} = 3600 \text{ s}$.

EXECUTE: $1.00 \text{ gigasecond} = (1.00\times10^9 \text{ s})\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ h}}\right)\left(\dfrac{1 \text{ y}}{365 \text{ days}}\right) = 31.7 \text{ y}.$

EVALUATE: The conversion $1 \text{ y} = 3.156\times10^7 \text{ s}$ assumes $1 \text{ y} = 365.24 \text{ d}$, which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

1.8. **IDENTIFY:** Apply the given conversion factors.

SET UP: $1 \text{ furlong} = 0.1250 \text{ mi}$ and $1 \text{ fortnight} = 14 \text{ days}$. $1 \text{ day} = 24 \text{ h}$.

EXECUTE: $(180{,}000 \text{ furlongs/fortnight})\left(\dfrac{0.125 \text{ mi}}{1 \text{ furlong}}\right)\left(\dfrac{1 \text{ fortnight}}{14 \text{ days}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ h}}\right) = 67 \text{ mi/h}$

EVALUATE: A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

1.9. **IDENTIFY:** Convert miles/gallon to km/L.

SET UP: $1 \text{ mi} = 1.609 \text{ km}$. $1 \text{ gallon} = 3.788 \text{ L}$.

EXECUTE: **(a)** $55.0 \text{ miles/gallon} = (55.0 \text{ miles/gallon})\left(\dfrac{1.609 \text{ km}}{1 \text{ mi}}\right)\left(\dfrac{1 \text{ gallon}}{3.788 \text{ L}}\right) = 23.4 \text{ km/L}.$

(b) The volume of gas required is $\dfrac{1500 \text{ km}}{23.4 \text{ km/L}} = 64.1 \text{ L}$. $\dfrac{64.1 \text{ L}}{45 \text{ L/tank}} = 1.4 \text{ tanks}.$

EVALUATE: $1 \text{ mi/gal} = 0.425 \text{ km/L}$. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so $1 \text{ mi/gal} \sim \frac{2}{4} \text{ km/L}$, which is roughly our result.

1.10. **IDENTIFY:** Convert units.

SET UP: Use the unit conversions given in the problem. Also, $100 \text{ cm} = 1 \text{ m}$ and $1000 \text{ g} = 1 \text{ kg}$.

EXECUTE: **(a)** $\left(60\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{5280 \text{ ft}}{1 \text{ mi}}\right) = 88\dfrac{\text{ft}}{\text{s}}$

(b) $\left(32\dfrac{\text{ft}}{\text{s}^2}\right)\left(\dfrac{30.48 \text{ cm}}{1 \text{ ft}}\right)\left(\dfrac{1 \text{ m}}{100 \text{ cm}}\right) = 9.8\dfrac{\text{m}}{\text{s}^2}$

(c) $\left(1.0\dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{100 \text{ cm}}{1 \text{ m}}\right)^3\left(\dfrac{1 \text{ kg}}{1000 \text{ g}}\right) = 10^3\dfrac{\text{kg}}{\text{m}^3}$

EVALUATE: The relations $60 \text{ mi/h} = 88 \text{ ft/s}$ and $1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$ are exact. The relation $32 \text{ ft/s}^2 = 9.8 \text{ m/s}^2$ is accurate to only two significant figures.

1.11. **IDENTIFY:** We know the density and mass; thus we can find the volume using the relation $\text{density} = \text{mass/volume} = m/V$. The radius is then found from the volume equation for a sphere and the result for the volume.

SET UP: $\text{Density} = 19.5 \text{ g/cm}^3$ and $m_{\text{critical}} = 60.0 \text{ kg}$. For a sphere $V = \frac{4}{3}\pi r^3$.

EXECUTE: $V = m_{\text{critical}}/\text{density} = \left(\dfrac{60.0 \text{ kg}}{19.5 \text{ g/cm}^3}\right)\left(\dfrac{1000 \text{ g}}{1.0 \text{ kg}}\right) = 3080 \text{ cm}^3.$

$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080\text{ cm}^3)} = 9.0\text{ cm}.$

EVALUATE: The density is very large, so the 130-pound sphere is small in size.

1.12. **IDENTIFY:** Convert units.

SET UP: We know the equalities $1\text{ mg} = 10^{-3}\text{ g}$, $1\text{ µg }10^{-6}\text{ g}$, and $1\text{ kg} = 10^3\text{ g}$.

EXECUTE: **(a)** $(410\text{ mg/day})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right)\left(\frac{1\,\mu\text{g}}{10^{-6}\text{ g}}\right) = 4.10\times10^5\,\mu\text{g/day}.$

(b) $(12\text{ mg/kg})(75\text{ kg}) = (900\text{ mg})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 0.900\text{ g}.$

(c) The mass of each tablet is $(2.0\text{ mg})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 2.0\times10^{-3}\text{ g}.$ The number of tablets required each day is the number of grams recommended per day divided by the number of grams per tablet:

$\frac{0.0030\text{ g/day}}{2.0\times10^{-3}\text{ g/tablet}} = 1.5$ tablet/day. Take 2 tablets each day.

(d) $(0.000070\text{ g/day})\left(\frac{1\text{ mg}}{10^{-3}\text{ g}}\right) = 0.070\text{ mg/day}.$

EVALUATE: Quantities in medicine and nutrition are frequently expressed in a wide variety of units.

1.13. **IDENTIFY:** Model the bacteria as spheres. Use the diameter to find the radius, then find the volume and surface area using the radius.

SET UP: From Appendix B, the volume V of a sphere in terms of its radius is $V = \frac{4}{3}\pi r^3$ while its surface area A is $A = 4\pi r^2$. The radius is one-half the diameter or $r = d/2 = 1.0\,\mu\text{m}$. Finally, the necessary equalities for this problem are: $1\,\mu\text{m} = 10^{-6}\text{ m}$; $1\text{ cm} = 10^{-2}\text{ m}$; and $1\text{ mm} = 10^{-3}\text{ m}$.

EXECUTE: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\,\mu\text{m})^3\left(\frac{10^{-6}\text{ m}}{1\,\mu\text{m}}\right)^3\left(\frac{1\text{ cm}}{10^{-2}\text{ m}}\right)^3 = 4.2\times10^{-12}\text{ cm}^3$ and

$$A = 4\pi r^2 = 4\pi(1.0\,\mu\text{m})^2\left(\frac{10^{-6}\text{ m}}{1\,\mu\text{m}}\right)^2\left(\frac{1\text{ mm}}{10^{-3}\text{ m}}\right)^2 = 1.3\times10^{-5}\text{ mm}^2$$

EVALUATE: On a human scale, the results are extremely small. This is reasonable because bacteria are not visible without a microscope.

1.14. **IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

SET UP: 12 mm has two significant figures and 5.98 mm has three significant figures.

EXECUTE: **(a)** $(12\text{ mm})\times(5.98\text{ mm}) = 72\text{ mm}^2$ (two significant figures)

(b) $\frac{5.98\text{ mm}}{12\text{ mm}} = 0.50$ (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

EVALUATE: The length of the rectangle is known only to the nearest mm, so the answers in parts **(c)** and (d) are known only to the nearest mm.

1.15. **IDENTIFY:** Use your calculator to display $\pi\times10^7$. Compare that number to the number of seconds in a year.

SET UP: $1\text{ yr} = 365.24$ days, $1\text{ day} = 24\text{ h}$, and $1\text{ h} = 3600\text{ s}$.

EXECUTE: $(365.24\text{ days/1 yr})\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 3.15567\ldots\times10^7\text{ s};\ \pi\times10^7\text{ s} = 3.14159\ldots\times10^7\text{ s}$

The approximate expression is accurate to two significant figures. The percent error is 0.45%.

EVALUATE: The close agreement is a numerical accident.

1.16. **IDENTIFY:** To asses the accuracy of the approximations, we must convert them to decimals.

SET UP: Use a calculator to calculate the decimal equivalent of each fraction and then round the numeral to the specified number of significant figures. Compare to π rounded to the same number of significant figures.

EXECUTE: **(a)** 22/7 = 3.14286 **(b)** 355/113 = 3.14159 **(c)** The exact value of π rounded to six significant figures is 3.14159.

EVALUATE: We see that 355/113 is a much better approximation to π than is 22/7.

1.17. **IDENTIFY:** Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.

SET UP: A mass of 1 kg is equivalent to a weight of about 2.2 lbs. $1\text{ in.} = 2.54\text{ cm}$. $1\text{ y} = 12\text{ months}$.

EXECUTE: **(a)** 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.

(b) $200\text{ m} = (2.00\times10^4\text{ cm})\left(\frac{1\text{ in.}}{2.54\text{ cm}}\right) = 7.9\times10^3$ inches. This is much greater than the height of a person.

(c) $200\text{ cm} = 2.00\text{ m} = 79\text{ inches} = 6.6\text{ ft}$. Some people are this tall, but not an ordinary man.

(d) $200\text{ mm} = 0.200\text{ m} = 7.9\text{ inches}$. This is much too short.

(e) $200\text{ months} = (200\text{ mon})\left(\frac{1\text{ y}}{12\text{ mon}}\right) = 17\text{ y}$. This is the age of a teenager; a middle-aged man is much older than this.

EVALUATE: None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

1.18. **IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

SET UP: Estimate 3×10^8 people, so 2×10^8 cars.

EXECUTE: $(\text{Number of cars}\times\text{miles/car day})/(\text{mi/gal}) = \text{gallons/day}$

$$(2\times10^8\text{ cars}\times10000\text{ mi/yr/car}\times1\text{ yr/365 days})/(20\text{ mi/gal}) = 3\times10^8\text{ gal/day}$$

EVALUATE: The number of gallons of gas used each day approximately equals the population of the U.S.

1.19. **IDENTIFY:** Estimate the number of blinks per minute. Convert minutes to years. Estimate the typical lifetime in years.

SET UP: Estimate that we blink 10 times per minute. $1\text{ y} = 365\text{ days}$. $1\text{ day} = 24\text{ h}$, $1\text{ h} = 60\text{ min}$. Use 80 years for the lifetime.

EXECUTE: The number of blinks is $(10\text{ per min})\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{365\text{ days}}{1\text{ y}}\right)(80\text{ y/lifetime}) = 4\times10^8$

EVALUATE: Our estimate of the number of blinks per minute can be off by a factor of two but our calculation is surely accurate to a power of 10.

1.20. **IDENTIFY:** Approximate the number of breaths per minute. Convert minutes to years and cm^3 to m^3 to find the volume in m^3 breathed in a year.

SET UP: Assume 10 breaths/min. $1\text{ y} = (365\text{ d})\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 5.3\times10^5\text{ min}$. $10^2\text{ cm} = 1\text{ m}$ so $10^6\text{ cm}^3 = 1\text{ m}^3$. The volume of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$, where r is the radius and d is the diameter. Don't forget to account for four astronauts.

EXECUTE: **(a)** The volume is $(4)(10\text{ breaths/min})(500\times10^{-6}\text{ m}^3)\left(\frac{5.3\times10^5\text{ min}}{1\text{ y}}\right) = 1\times10^4\text{ m}^3\text{/yr}$.

(b) $d = \left(\dfrac{6V}{\pi}\right)^{1/3} = \left(\dfrac{6[1\times10^4\ \text{m}^3]}{\pi}\right)^{1/3} = 27\ \text{m}$

EVALUATE: Our estimate assumes that each cm^3 of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

1.21. **IDENTIFY:** Estimation problem.

SET UP: Estimate that the pile is $18\ \text{in.}\times 18\ \text{in.}\times 5\ \text{ft}\ 8\ \text{in.}$. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

EXECUTE: The volume of gold in the pile is $V = 18\ \text{in.}\times 18\ \text{in.}\times 68\ \text{in.} = 22{,}000\ \text{in.}^3$. Convert to cm^3:

$$V = 22{,}000\ \text{in.}^3(1000\ \text{cm}^3/61.02\ \text{in.}^3) = 3.6\times10^5\ \text{cm}^3.$$

The density of gold is $19.3\ \text{g/cm}^3$, so the mass of this volume of gold is

$$m = (19.3\ \text{g/cm}^3)(3.6\times10^5\ \text{cm}^3) = 7\times10^6\ \text{g}.$$

The monetary value of one gram is \$10, so the gold has a value of $(\$10/\text{gram})(7\times10^6\ \text{grams}) = \$7\times10^7$, or about $\$100\times10^6$ (one hundred million dollars).

EVALUATE: This is quite a large pile of gold, so such a large monetary value is reasonable.

1.22. **IDENTIFY:** Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

SET UP: An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

EXECUTE: $N_{\text{beats}} = (75\ \text{beats/min})\left(\dfrac{60\ \text{min}}{1\ \text{h}}\right)\left(\dfrac{24\ \text{h}}{1\ \text{day}}\right)\left(\dfrac{365\ \text{days}}{\text{yr}}\right)\left(\dfrac{80\ \text{yr}}{\text{lifespan}}\right) = 3\times10^9\ \text{beats/lifespan}$

$V_{\text{blood}} = (50\ \text{cm}^3/\text{beat})\left(\dfrac{1\ \text{L}}{1000\ \text{cm}^3}\right)\left(\dfrac{1\ \text{gal}}{3.788\ \text{L}}\right)\left(\dfrac{3\times10^9\ \text{beats}}{\text{lifespan}}\right) = 4\times10^7\ \text{gal/lifespan}$

EVALUATE: This is a very large volume.

1.23. **IDENTIFY:** Estimate the diameter of a drop and from that calculate the volume of a drop, in m^3. Convert m^3 to L.

SET UP: Estimate the diameter of a drop to be $d = 2\ \text{mm}$. The volume of a spherical drop is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. $10^3\ \text{cm}^3 = 1\ \text{L}$.

EXECUTE: $V = \frac{1}{6}\pi(0.2\ \text{cm})^3 = 4\times10^{-3}\ \text{cm}^3$. The number of drops in 1.0 L is $\dfrac{1000\ \text{cm}^3}{4\times10^{-3}\ \text{cm}^3} = 2\times10^5$

EVALUATE: Since $V \sim d^3$, if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

1.24. **IDENTIFY:** Draw the vector addition diagram to scale.

SET UP: The two vectors $\vec{A}$ and $\vec{B}$ are specified in the figure that accompanies the problem.

EXECUTE: **(a)** The diagram for $\vec{R} = \vec{A} + \vec{B}$ is given in Figure 1.24a. Measuring the length and angle of $\vec{R}$ gives $R = 9.0$ m and an angle of $\theta = 34°$.

(b) The diagram for $\vec{E} = \vec{A} - \vec{B}$ is given in Figure 1.24b. Measuring the length and angle of $\vec{E}$ gives $D = 22$ m and an angle of $\theta = 250°$.

(c) $-\vec{A} - \vec{B} = -(\vec{A} + \vec{B})$, so $-\vec{A} - \vec{B}$ has a magnitude of 9.0 m (the same as $\vec{A} + \vec{B}$) and an angle with the $+x$ axis of $214°$ (opposite to the direction of $\vec{A} + \vec{B}$).

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has a magnitude of 22 m and an angle with the $+x$ axis of $70°$ (opposite to the direction of $\vec{A} - \vec{B}$).

EVALUATE: The vector $-\vec{A}$ is equal in magnitude and opposite in direction to the vector $\vec{A}$.

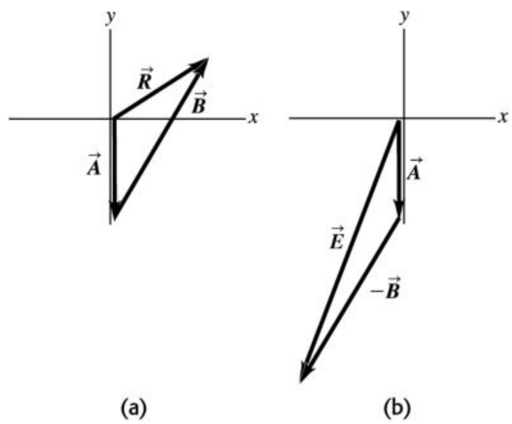


Figure 1.24

1.25. **IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.

SET UP: Call the three displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$. The resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

EXECUTE: The vector addition diagram is given in Figure 1.25. Careful measurement gives that $\vec{R}$ is 7.8 km, 38° north of east.

EVALUATE: The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements, $2.6\text{ km} + 4.0\text{ km} + 3.1\text{ km}$.

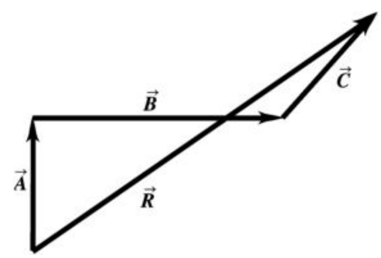


Figure 1.25

1.26. **IDENTIFY:** Since she returns to the starting point, the vector sum of the four displacements must be zero.

SET UP: Call the three given displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$, and call the fourth displacement $\vec{D}$. $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$.

EXECUTE: The vector addition diagram is sketched in Figure 1.26. Careful measurement gives that $\vec{D}$ is 144 m, 41° south of west.

EVALUATE: $\vec{D}$ is equal in magnitude and opposite in direction to the sum $\vec{A} + \vec{B} + \vec{C}$.

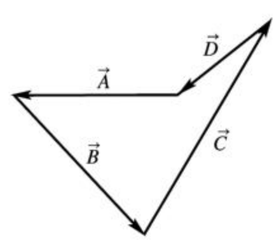


Figure 1.26

1.27. **IDENTIFY:** For each vector $\vec{V}$, use that $V_x = V\cos\theta$ and $V_y = V\sin\theta$, when θ is the angle $\vec{V}$ makes with the $+x$ axis, measured counterclockwise from the axis.

SET UP: For $\vec{A}$, $\theta = 270.0°$. For $\vec{B}$, $\theta = 60.0°$. For $\vec{C}$, $\theta = 205.0°$. For $\vec{D}$, $\theta = 143.0°$.

EXECUTE: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EVALUATE: The signs of the components correspond to the quadrant in which the vector lies.

1.28. **IDENTIFY:** $\tan\theta = \dfrac{A_y}{A_x}$, for θ measured counterclockwise from the $+x$-axis.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE:

(a) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{2.00\text{ m}} = -0.500.$ $\theta = \tan^{-1}(-0.500) = 360° - 26.6° = 333°.$

(b) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{2.00\text{ m}} = 0.500.$ $\theta = \tan^{-1}(0.500) = 26.6°.$

(c) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{-2.00\text{ m}} = -0.500.$ $\theta = \tan^{-1}(-0.500) = 180° - 26.6° = 153°.$

(d) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{-2.00\text{ m}} = 0.500.$ $\theta = \tan^{-1}(0.500) = 180° + 26.6° = 207°$

EVALUATE: The angles 26.6° and 207° have the same tangent. Our sketch tells us which is the correct value of θ.

1.29. **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: **(a)** $\tan 32.0° = \dfrac{|A_x|}{|A_y|}$

$|A_x| = (9.60\text{ m})\tan 32.0° = 6.00\text{ m}.$ $A_x = -6.00$ m.

(b) $A = \sqrt{A_x^2 + A_y^2} = 11.3$ m.

EVALUATE: The magnitude is greater than either of the components.

1.30. **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

SET UP: Use the tangent of the given angle and the definition of vector magnitude.

EXECUTE: **(a)** $\tan 34.0° = \dfrac{|A_x|}{|A_y|}$

$|A_y| = \dfrac{|A_x|}{\tan 34.0°} = \dfrac{16.0\text{ m}}{\tan 34.0°} = 23.72\text{ m}$

$A_y = -23.7$ m.

(b) $A = \sqrt{A_x^2 + A_y^2} = 28.6$ m.

EVALUATE: The magnitude is greater than either of the components.

1.31. **IDENTIFY:** If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use C_x and C_y to find the magnitude and direction of $\vec{C}$.

SET UP: From Figure E1.24 in the textbook, $A_x = 0$, $A_y = -8.00$ m and $B_x = +B\sin 30.0° = 7.50$ m, $B_y = +B\cos 30.0° = 13.0$ m.

EXECUTE: **(a)** $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50$ m and $C_y = A_y + B_y = +5.00$ m. $C = 9.01$ m.

$\tan\theta = \dfrac{C_y}{C_x} = \dfrac{5.00 \text{ m}}{7.50 \text{ m}}$ and $\theta = 33.7°$.

(b) $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by $33.7°$.

(c) $\vec{D} = \vec{A} - \vec{B}$ so $D_x = A_x - B_x = -7.50$ m and $D_y = A_y - B_y = -21.0$ m. $D = 22.3$ m.

$\tan\phi = \dfrac{D_y}{D_x} = \dfrac{-21.0 \text{ m}}{-7.50 \text{ m}}$ and $\phi = 70.3°$. $\vec{D}$ is in the 3rd quadrant and the angle θ counterclockwise from the $+x$ axis is $180° + 70.3° = 250.3°$.

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has magnitude 22.3 m and direction specified by $\theta = 70.3°$.

EVALUATE: These results agree with those calculated from a scale drawing in Problem 1.24.

1.32. **IDENTIFY:** Find the vector sum of the three given displacements.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are:

$\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east.

EXECUTE: $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km})\cos(45°) = 6.2$ km; $R_y = A_y + B_y + C_y = 2.6 \text{ km} + 0 + (3.1 \text{ km})(\sin 45°) = 4.8$ km; $R = \sqrt{R_x^2 + R_y^2} = 7.8$ km; $\theta = \tan^{-1}[(4.8 \text{ km})/(6.2 \text{ km})] = 38°$;

$\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.32.

EVALUATE: Both R_x and R_y are positive and $\vec{R}$ is in the first quadrant.

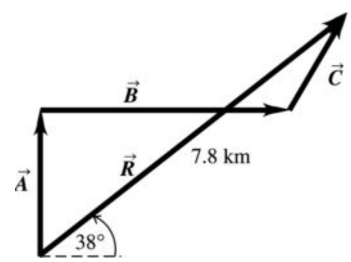
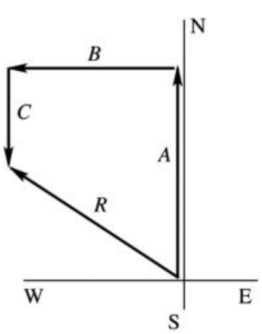


Figure 1.32

1.33. **IDENTIFY:** Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

SET UP:



$A = 3.25$ km

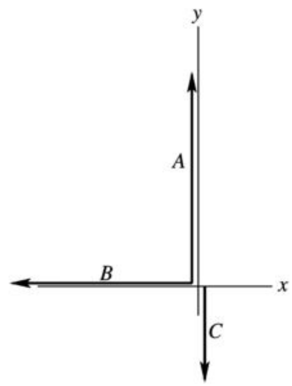
$B = 2.20$ km

$C = 1.50$ km

Figure 1.33a

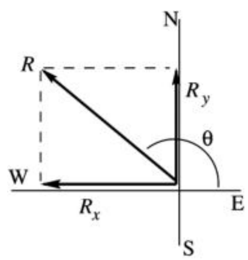
Select a coordinate system where $+x$ is east and $+y$ is north. Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements of the professor. Then the resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. By the method of components, $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$. Find the *x* and *y* components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its *x* and *y* components that we have calculated. As always it is essential to draw a sketch.

EXECUTE:



$$A_x = 0,\ A_y = +3.25\text{ km}$$
$$B_x = -2.20\text{ km},\ B_y = 0$$
$$C_x = 0,\ C_y = -1.50\text{ km}$$
$$R_x = A_x + B_x + C_x$$
$$R_x = 0 - 2.20\text{ km} + 0 = -2.20\text{ km}$$
$$R_y = A_y + B_y + C_y$$
$$R_y = 3.25\text{ km} + 0 - 1.50\text{ km} = 1.75\text{ km}$$

Figure 1.33b



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20\text{ km})^2 + (1.75\text{ km})^2}$$
$$R = 2.81\text{ km}$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{1.75\text{ km}}{-2.20\text{ km}} = -0.800$$
$$\theta = 141.5°$$

Figure 1.33c

The angle θ measured counterclockwise from the $+x$-axis. In terms of compass directions, the resultant displacement is $38.5°$ N of W.

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant. This agrees with the vector addition diagram.

1.34. **IDENTIFY:** Use $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$ to calculate the magnitude and direction of each of the given vectors.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE: **(a)** $\sqrt{(-8.60\text{ cm})^2 + (5.20\text{ cm})^2} = 10.0\text{ cm}$, $\arctan\left(\dfrac{5.20}{-8.60}\right) = 148.8°$ (which is $180° - 31.2°$).

(b) $\sqrt{(-9.7\text{ m})^2 + (-2.45\text{ m})^2} = 10.0\text{ m}$, $\arctan\left(\dfrac{-2.45}{-9.7}\right) = 14° + 180° = 194°$.

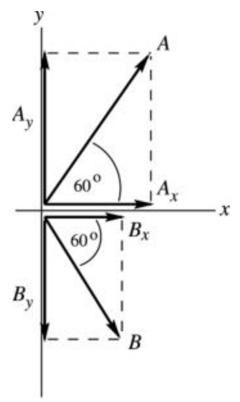
(c) $\sqrt{(7.75\text{ km})^2 + (-2.70\text{ km})^2} = 8.21\text{ km}$, $\arctan\left(\dfrac{-2.7}{7.75}\right) = 340.8°$ (which is $360° - 19.2°$).

EVALUATE: In each case the angle is measured counterclockwise from the $+x$ axis. Our results for θ agree with our sketches.

1.35. **IDENTIFY:** Vector addition problem. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

SET UP: Find the x- and y-components of $\vec{A}$ and $\vec{B}$. Then the x- and y-components of the vector sum are calculated from the x- and y-components of $\vec{A}$ and $\vec{B}$.

EXECUTE:

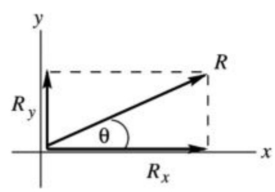


$A_x = A\cos(60.0°)$
$A_x = (2.80\text{ cm})\cos(60.0°) = +1.40\text{ cm}$
$A_y = A\sin(60.0°)$
$A_y = (2.80\text{ cm})\sin(60.0°) = +2.425\text{ cm}$
$B_x = B\cos(-60.0°)$
$B_x = (1.90\text{ cm})\cos(-60.0°) = +0.95\text{ cm}$
$B_y = B\sin(-60.0°)$
$B_y = (1.90\text{ cm})\sin(-60.0°) = -1.645\text{ cm}$

Note that the signs of the components correspond to the directions of the component vectors.

Figure 1.35a

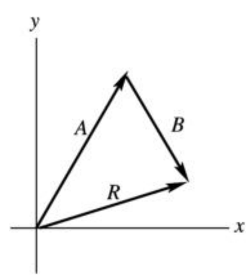
(a) Now let $\vec{R} = \vec{A} + \vec{B}$.
$R_x = A_x + B_x = +1.40\text{ cm} + 0.95\text{ cm} = +2.35\text{ cm}.$
$R_y = A_y + B_y = +2.425\text{ cm} - 1.645\text{ cm} = +0.78\text{ cm}.$



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35\text{ cm})^2 + (0.78\text{ cm})^2}$
$R = 2.48\text{ cm}$
$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{+0.78\text{ cm}}{+2.35\text{ cm}} = +0.3319$
$\theta = 18.4°$

Figure 1.35b

EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is



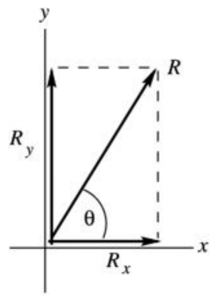
$\vec{R}$ is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

Figure 1.35c

(b) EXECUTE: Now let $\vec{R} = \vec{A} - \vec{B}$.

$R_x = A_x - B_x = +1.40\text{ cm} - 0.95\text{ cm} = +0.45\text{ cm}.$

$R_y = A_y - B_y = +2.425\text{ cm} + 1.645\text{ cm} = +4.070\text{ cm}.$



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45\text{ cm})^2 + (4.070\text{ cm})^2}$

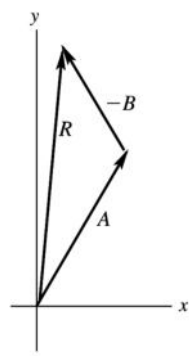
$R = 4.09\text{ cm}$

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{4.070\text{ cm}}{0.45\text{ cm}} = +9.044$

$\theta = 83.7°$

Figure 1.35d

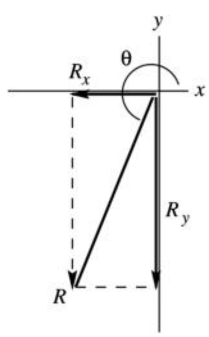
EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + (-\vec{B})$ is



$\vec{R}$ is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.35e

(c) EXECUTE:



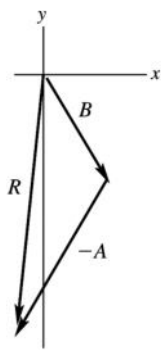
$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$R = 4.09\text{ cm}$ and $\theta = 83.7° + 180° = 264°$

Figure 1.35f

EVALUATE: The vector addition diagram for $\vec{R} = \vec{B} + (-\vec{A})$ is



$\vec{R}$ is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.35g

1.36. **IDENTIFY:** The general expression for a vector written in terms of components and unit vectors is $\vec{A} = A_x\hat{i} + A_y\hat{j}$.

SET UP: $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\vec{i} - 30\vec{j}$

EXECUTE: **(a)** $A_x = 5.0,\ A_y = -6.3$ **(b)** $A_x = 11.2,\ A_y = -9.91$ **(c)** $A_x = -15.0,\ A_y = 22.4$

(d) $A_x = 20,\ A_y = -30$

EVALUATE: The components are signed scalars.

1.37. **IDENTIFY:** Find the components of each vector and then use the general equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ for a vector in terms of its components and unit vectors.

SET UP: $A_x = 0,\ A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EXECUTE: $\vec{A} = (-8.00 \text{ m})\hat{j}$; $\vec{B} = (7.50 \text{ m})\hat{i} + (13.0 \text{ m})\hat{j}$; $\vec{C} = (-10.9 \text{ m})\hat{i} + (-5.07 \text{ m})\hat{j}$;

$\vec{D} = (-7.99 \text{ m})\hat{i} + (6.02 \text{ m})\hat{j}$.

EVALUATE: All these vectors lie in the *xy*-plane and have no *z*-component.

1.38. **IDENTIFY:** Find A and B. Find the vector difference using components.

SET UP: Identify the *x*- and *y*-components and use $A = \sqrt{A_x^2 + A_y^2}$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$; $A_x = +4.00$; $A_y = +7.00$.

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (7.00)^2} = 8.06$. $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$;

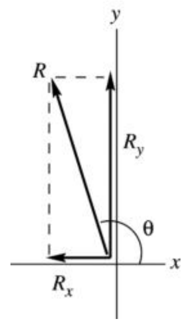
$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$.

EVALUATE: Note that the magnitudes of $\vec{A}$ and $\vec{B}$ are each larger than either of their components.

EXECUTE: **(b)** $\vec{A} - \vec{B} = 4.00\hat{i} + 7.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (7.00 + 2.00)\hat{j}$.

$\vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$

(c) Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$. Then $R_x = -1.00,\ R_y = 9.00$.



$R=\sqrt{R_x^2+R_y^2}$

$R=\sqrt{(-1.00)^2+(9.00)^2}=9.06.$

$\tan\theta=\dfrac{R_y}{R_x}=\dfrac{9.00}{-1.00}=-9.00$

$\theta=-83.6°+180°=96.3°.$

Figure 1.38

EVALUATE: $R_x<0$ and $R_y>0$, so $\vec{R}$ is in the 2nd quadrant.

1.39. **IDENTIFY:** Use trigonometry to find the components of each vector. Use $R_x = A_x + B_x + \cdots$ and $R_y = A_y + B_y + \cdots$ to find the components of the vector sum. The equation $\vec{A}=A_x\hat{i}+A_y\hat{j}$ expresses a vector in terms of its components.

SET UP: Use the coordinates in the figure that accompanies the problem.

EXECUTE: **(a)** $\vec{A}=(3.60\text{ m})\cos 70.0°\hat{i}+(3.60\text{ m})\sin 70.0°\hat{j}=(1.23\text{ m})\hat{i}+(3.38\text{ m})\hat{j}$

$\vec{B}=-(2.40\text{ m})\cos 30.0°\hat{i}-(2.40\text{ m})\sin 30.0°\hat{j}=(-2.08\text{ m})\hat{i}+(-1.20\text{ m})\hat{j}$

(b) $\vec{C}=(3.00)\,\vec{A}-(4.00)\,\vec{B}=(3.00)(1.23\text{ m})\hat{i}+(3.00)(3.38\text{ m})\hat{j}-(4.00)(-2.08\text{ m})\hat{i}-(4.00)(-1.20\text{ m})\hat{j}$

$\vec{C}=(12.01\text{ m})\hat{i}+(14.94\text{ m})\hat{j}$

(c) From $A=\sqrt{A_x^2+A_y^2}$ and $\tan\theta=\dfrac{A_y}{A_x}$,

$C=\sqrt{(12.01\text{ m})^2+(14.94\text{ m})^2}=19.17\text{ m},\ \arctan\left(\dfrac{14.94\text{ m}}{12.01\text{ m}}\right)=51.2°$

EVALUATE: C_x and C_y are both positive, so θ is in the first quadrant.

1.40. **IDENTIFY:** We use the vector components and trigonometry to find the angles.

SET UP: Use the fact that $\tan\theta = A_y / A_x$.

EXECUTE: **(a)** $\tan\theta = A_y / A_x = \dfrac{6.00}{-3.00}$. θ = 117° with the +*x*-axis.

(b) $\tan\theta = B_y / B_x = \dfrac{2.00}{7.00}$. θ = 15.9°.

(c) First find the components of $\vec{C}$. $C_x = A_x + B_x = -3.00 + 7.00 = 4.00$,

$C_y = A_y + B_y = 6.00 + 2.00 = 8.00$

$\tan\theta = C_y / C_x = \dfrac{8.00}{4.00} = 2.00$. θ = 63.4°

EVALUATE: Sketching each of the three vectors to scale will show that the answers are reasonable.

1.41. **IDENTIFY:** $\vec{A}$ and $\vec{B}$ are given in unit vector form. Find *A*, *B* and the vector difference $\vec{A}-\vec{B}$.

SET UP: $\vec{A}=-2.00\vec{i}+3.00\vec{j}+4.00\vec{k},\ \vec{B}=3.00\vec{i}+1.00\vec{j}-3.00\vec{k}$

Use $A=\sqrt{A_x^2+A_y^2+A_z^2}$ to find the magnitudes of the vectors.

EXECUTE: **(a)** $A=\sqrt{A_x^2+A_y^2+A_z^2}=\sqrt{(-2.00)^2+(3.00)^2+(4.00)^2}=5.38$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$

(b) $\vec{A} - \vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$\vec{A} - \vec{B} = (-2.00 - 3.00)\hat{i} + (3.00 - 1.00)\hat{j} + (4.00 - (-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}.$

(c) Let $\vec{C} = \vec{A} - \vec{B}$, so $C_x = -5.00,\ C_y = +2.00,\ C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$ have the same magnitude but opposite directions.

EVALUATE: A, B, and C are each larger than any of their components.

1.42. **IDENTIFY:** Target variables are $\vec{A} \cdot \vec{B}$ and the angle ϕ between the two vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the scalar product using $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. The angle ϕ can then be found from $\vec{A} \cdot \vec{B} = AB\cos\phi$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j},\ \vec{B} = 5.00\hat{i} - 2.00\hat{j};\ A = 8.06,\ B = 5.39.$

$\vec{A} \cdot \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \cdot (5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (7.00)(-2.00) = 20.0 - 14.0 = +6.00.$

(b) $\cos\phi = \dfrac{\vec{A} \cdot \vec{B}}{AB} = \dfrac{6.00}{(8.06)(5.39)} = 0.1382;\ \phi = 82.1°.$

EVALUATE: The component of $\vec{B}$ along $\vec{A}$ is in the same direction as $\vec{A}$, so the scalar product is positive and the angle ϕ is less than $90°$.

1.43. **IDENTIFY:** $\vec{A} \cdot \vec{B} = AB\cos\phi$

SET UP: For $\vec{A}$ and $\vec{B}$, $\phi = 150.0°$. For $\vec{B}$ and $\vec{C}$, $\phi = 145.0°$. For $\vec{A}$ and $\vec{C}$, $\phi = 65.0°$.

EXECUTE: **(a)** $\vec{A} \cdot \vec{B} = (8.00\ \text{m})(15.0\ \text{m})\cos 150.0° = -104\ \text{m}^2$

(b) $\vec{B} \cdot \vec{C} = (15.0\ \text{m})(12.0\ \text{m})\cos 145.0° = -148\ \text{m}^2$

(c) $\vec{A} \cdot \vec{C} = (8.00\ \text{m})(12.0\ \text{m})\cos 65.0° = 40.6\ \text{m}^2$

EVALUATE: When $\phi < 90°$ the scalar product is positive and when $\phi > 90°$ the scalar product is negative.

1.44. **IDENTIFY:** Target variable is the vector $\vec{A} \times \vec{B}$ expressed in terms of unit vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, and $\hat{j} \times \hat{i} = -\hat{k}$.

EXECUTE: $\vec{A} = 4.00\hat{i} + 7.00\hat{j},\ \ \vec{B} = 5.00\hat{i} - 2.00\hat{j}.$

$\vec{A} \times \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \times (5.00\hat{i} - 2.00\hat{j}) = 20.0\hat{i} \times \hat{i} - 8.00\hat{i} \times \hat{j} + 35.0\hat{j} \times \hat{i} - 14.0\hat{j} \times \hat{j}.$ But $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$ and $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{i} = -\hat{k}$, so $\vec{A} \times \vec{B} = -8.00\hat{k} + 35.0(-\hat{k}) = -43.0\hat{k}.$ The magnitude of $\vec{A} \times \vec{B}$ is 43.0.

EVALUATE: Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the xy-plane is in the plane of the paper and the z-axis is directed out toward you. By the right-hand rule $\vec{A} \times \vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

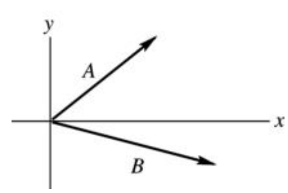


Figure 1.44

1.45. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining $\vec{A}\cdot\vec{B} = AB\cos\phi$ and $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$, to give the angle ϕ as $\phi = \arccos\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\dfrac{A_xB_x + A_yB_y}{AB}\right)$.

SET UP: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22,\ A = \sqrt{40},\ B = \sqrt{13},$ and so $\phi = \arccos\left(\dfrac{-22}{\sqrt{40}\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60,\ A = \sqrt{34},\ B = \sqrt{136},\ \ \phi = \arccos\left(\dfrac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0,\ 0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0,\ 90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0,\ \phi = 90°$ and the two vectors are perpendicular.

1.46. **IDENTIFY:** The right-hand rule gives the direction and $|\vec{A}\times\vec{B}| = AB\sin\phi$ gives the magnitude.

SET UP: $\phi = 120.0°$.

EXECUTE: **(a)** The direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is $AB\sin\phi = (2.80\text{ cm})(1.90\text{ cm})\sin 120° = 4.61\text{ cm}^2$.

(b) Rather than repeat the calculations, $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$ may be used to see that $\vec{B}\times\vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

EVALUATE: For part (a) we could use the components of the cross product and note that the only non-vanishing component is $C_z = A_x B_y - A_y B_x = (2.80\text{ cm})\cos 60.0°(-1.90\text{ cm})\sin 60°$

$$-(2.80\text{ cm})\sin 60.0°(1.90\text{ cm})\cos 60.0° = -4.61\text{ cm}^2.$$

This gives the same result.

1.47. **IDENTIFY:** $\vec{A}\times\vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

SET UP: $\phi = 180° - 53° = 127°$

EXECUTE: **(a)** $|\vec{A}\times\vec{D}| = (8.00\text{ m})(10.0\text{ m})\sin 127° = 63.9\text{ m}^2$. The right-hand rule says $\vec{A}\times\vec{D}$ is in the $-z$-direction (into the page).

(b) $\vec{D}\times\vec{A}$ has the same magnitude as $\vec{A}\times\vec{D}$ and is in the opposite direction.

EVALUATE: The component of $\vec{D}$ perpendicular to $\vec{A}$ is $D_\perp = D\sin 53.0° = 7.99$ m.

$|\vec{A}\times\vec{D}| = AD_\perp = 63.9\text{ m}^2$, which agrees with our previous result.

1.48. **IDENTIFY:** Apply Eqs. (1.16) and (1.20).

SET UP: The angle between the vectors is $20° + 90° + 30° = 140°$.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = AB\cos\phi$ gives $\vec{A}\cdot\vec{B} = (3.60\text{ m})(2.40\text{ m})\cos 140° = -6.62\text{ m}^2$.

(b) From $|\vec{A}\times\vec{B}| = AB\sin\phi$, the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin 140° = 5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction).

EVALUATE: We could also use $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ and the cross product, with the components of $\vec{A}$ and $\vec{B}$.

1.49. **IDENTIFY:** We model the earth, white dwarf, and neutron star as spheres. Density is mass divided by volume.

SET UP: We know that density = mass/volume = m/V where $V = \frac{4}{3}\pi r^3$ for a sphere. From Appendix B, the earth has mass of $m = 5.97\times10^{24}$ kg and a radius of $r = 6.37\times10^{6}$ m whereas for the sun at the end of its lifetime, $m = 1.99\times10^{30}$ kg and r = 7500 km = 7.5×10^{6} m. The star possesses a radius of r = 10 km = 1.0×10^{4} m and a mass of $m = 1.99\times10^{30}$ kg.