# FOUNDATIONS

# 1

*Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems*

## Developing a Feel

**1.** $10^{-2}$ m **2.** $10^{2}$ m **3.** $10^{2}$ m **4.** $10^{1}$ $m^{2}$ **5.** $10^{3}$ **6.** $10^{3}$ kg **7.** $10^{4}$ kg **8.** $10^{13}$ kg **9.** $10^{5}$ kg **10.** $10^{11}$ **11.** $10^{9}$ **12.** $10^{5}$

## Guided Problems

### 1.2 Solar oxygen

**1. Getting Started** Much of the plan used in Worked Problem 1.1 can be used here. We wish to use the volume of the sun and the percent of the Sun that is made up of oxygen to calculate the mass density and number density of oxygen atoms. As before we assume the Sun is a perfect sphere.

**2. Devise Plan** As before, we use the mass density $\rho = m/V$ and the number density $n = N/V$. We will use $V = \frac{4}{3}\pi R^3$ for the volume of the Sun. We will use the total mass of the Sun and the fraction of that mass that is made up of oxygen to calculate the mass of oxygen in the Sun. We will divide by volume to get the mass density. Then, one need only divide by the mass of a single oxygen atom to convert from mass density to number density.

**3. Execute Plan** First, the mass density is given by

$$\rho = \frac{m}{V} = \frac{m_{\text{oxygen}}}{\frac{4}{3}\pi R_{\text{Sun}}^3} = \frac{(0.00970)M_{\text{Sun}}}{\frac{4}{3}\pi R_{\text{Sun}}^3}$$

$$\rho = \frac{(0.00970)(1.99\times10^{30}\ \text{kg})}{\frac{4}{3}\pi(6.96\times10^{8}\ \text{m})^3} = 13.7\ \text{kg/m}^3$$

We convert from mass density to number density using

$$n = \frac{\rho}{m_{\text{atom}}} = \frac{13.7\ \text{kg/m}^3}{2.66\times10^{-26}\ \text{kg/atom}} = 5.14\times10^{26}\ \text{atoms/m}^3$$

**4. Evaluate Result** The calculated mass density of oxygen is approximately two orders of magnitude smaller than the mass density of hydrogen calculated in Worked Problem 1.1. This is what we expect, because the oxygen accounts for only about 1% of the mass of the Sun and hydrogen makes up approximately 70%. Since the oxygen accounts for two orders of magnitude less mass, it is perfectly sensible that the mass density of oxygen would also be two orders of magnitude smaller. Similarly, the number density of oxygen is about three orders of magnitude smaller than the number density of hydrogen. If the only issue were the relative percentages of the Sun's mass made up by oxygen and hydrogen, we would expect a difference of only two orders of magnitude. However, oxygen is also much more massive than hydrogen (around one order of magnitude more massive). Hence, mass densities that differ by two orders of magnitude mean number densities that differ by three orders of magnitude. Our results are consistent with those of Worked Problem 1.1.

## 1.4 Box volume

**1. Getting Started** This problem is similar to Worked Problem 1.3 in that we are given a mixture of units which we must convert to SI units. However, the expressions for volume here is completely different from that of Worked Problem 1.3.

**2. Devise Plan** We convert from feet to inches using the conversion factor $1 \text{ ft} = 12 \text{ in},$ and convert from inches to meters using $1 \text{ in} = 0.0254 \text{ m}.$ We can convert millimeters and centimeters to meters by simply dividing by the appropriate factor of ten. When all quantities are in units of meters, we proceed to find the volume of the box using $V_{\text{box}} = \ell w h,$ where $\ell,$ $w,$ and $h$ are the length, width, and height of the box, respectively.

**3. Execute Plan** We first convert the length, width and height to units of meters:

$$\ell = 1420 \text{ mm} \times \frac{1 \text{ m}}{10^3 \text{ mm}} = 1.420 \text{ m}$$

$$w = 2.75 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{0.0254 \text{ m}}{1 \text{ in}} = 0.838 \text{ m}$$

$$h = 87.8 \text{ cm} \times \frac{1 \text{ m}}{10^2 \text{ cm}} = 0.878 \text{ m}$$

Finally the volume is given by $V_{\text{box}} = \ell w h = (1.420 \text{ m})(0.838 \text{ m})(0.878 \text{ m}) = 1.05 \text{ m}^3.$

**4. Evaluate Result** A volume of $1.05 \text{ m}^3$ is reasonable. The box had a size that was on the order of 1.0 m in each dimension. One dimension had a length greater than 1.0 m and the other two were smaller than 1.0 m. This is the approximate size we would expect.

## 1.6 Digits on your own

**1. Getting Started** The methods used in Worked Problem 1.5 are essentially the same that will be used here. The expressions involve quantities with varying numbers of significant digits. The number of significant digits in the answer will depend on how many significant digits are in the given in the problem statement, and on the operation carried out between quantities (subtraction, multiplication, etc).

**2. Devise Plan** As in Worked Problem 1.5, the number of significant digits in a product or quotient is the same as the number of significant digits in the input quantity that has the *fewest* significant digits. The number of decimal places in a sum or difference is the same as the number of decimal places in the input quantity that has the *fewest* decimal places. To express our answers as orders of magnitude, we write each in scientific notation, round the coefficient either down to 1 (for coefficients $\leq 3$) or up to 10 (for coefficients $> 3$). We then write the answer as a power of ten without the coefficient.

**3. Execute Plan** (*a*) In $(205)(0.0041)(489.62),$ the middle quantity $(0.0041)$ has the fewest significant digits, with only two. Hence the answer must be given to only two significant digits: $4.1 \times 10^2.$ Here the coefficient is greater than 3, so we round it to 10 and obtain for the order of magnitude $10 \times 10^2 = 10^3.$ (*b*) Here, the first factor is given to four significant digits, which is the fewest of all factors (since $\pi$ is known to many digits). Hence the answer must be given to five significant digits: $2.475 \times 10^1.$ Here the prefactor is less than 3, so we round it down to 1. This yields an order of magnitude of $1 \times 10 = 10^1.$ (*c*) The first term is known to the thousandths place, whereas the second term is known only to the tenths place. Hence the answer can be given only out to the tenths place: $6.9802 \times 10^3.$ The prefactor is greater than 3, so we round it to 10. This yields an order of magnitude of $10 \times 10^3 = 10^4.$

**4. Evaluate Result** We could check our answers by getting order of magnitude estimates for each number. (*a*) Using orders of magnitude of $(205)(0.0041)(489.62)$ yields $(10^2)(10^{-2})(10^3) = 10^3$, which is consistent with our answer. (*b*) Similarly, $(190.8)(0.407\ 500)/\pi$ becomes $(10^2)(10^0)/(10^1) = 10^1$, which is again consistent with our answer. (*c*) Finally, to nearest order $(6980.035) + (0.2)$ yields $(10^4) - (10^{-1}) = 10^4$, which is also consistent.

## 1.8 Roof area

**1. Getting Started** We will approximate the United States as a rectangle 5,000 km wide and 3,000 km high. A very small percentage of this area is occupied by buildings. Only in cities can we approximate the surface as being covered by structures.

**2. Devise Plan** The approximate area of the United States is $(10^4\ \text{km})(10^3\ \text{km}) = 10^7\ \text{km}^2$. Depending on one's definition of a "city" the percentage of this surface area that is occupied by cities could be anywhere between 0.1% and 1%. But since even cities are not completely filled with structures, we will use the smaller of these two percentages for our estimate (0.1%). These two numbers can be used to find an order of magnitude estimate of the total roof area in the United States.

**3. Execute Plan** The roof area is given by $A_{\text{roof}} = (0.1\%)A_{\text{US}} = (0.001)(10^7\ \text{km}^2) = 10^4\ \text{km}^2$.

**4. Evaluate Result** We could check our result by estimating the roof surface area in a completely different way. Let us start with the population of the United States ($3\times10^8$ people to one significant digit), and assume that there exists a structure that houses every set of three or four people, and assume further that these structures have a footprint of approximately $30\ \text{m}^2$. Of course, many people live alone and many people live in apartment buildings that have many floors (meaning less of a footprint per person). But these cases might cancel each other out, making our estimate plausible. This yields a surface area of $2.3\times10^9\ \text{m}^2$. Let us double this number to account for the places of business that employ many of these people. This gives a total surface area of order $10^{10}\ \text{m}^2$. Using known conversion factors, one can see that this is equivalent to $10^4\ \text{km}^2$. This agrees with our previous estimate.

# Questions and Problems

**1.1.** The word "undetectable" prevents this from being a valid scientific hypothesis. A hypothesis must be experimentally verifiable.

**1.2.** You assume that the competing product contains non-zero fat, and that the serving sizes of the two are equal. Say the two foods contain the same amount of fat per ounce. The maker of the product being advertised could print his label showing a *recommended serving size* 50 percent smaller than the recommended serving size of the competing product. This makes the claim of 50 percent less fat *per serving* true but of course misleading.

**1.3.** You assume the sequence is linear, meaning each entry is larger than the previous one by a constant amount. As an alternative, the sequence could be formed by starting with 1, 2 and then setting the $n^{\text{th}}$ term $c_n$ equal to the sum of the previous two terms: $c_n = c_{n-1} + c_{n-2}$. This would work for $c_3 = c_2 + c_1 = 2 + 1 = 3$, and would yield 5 as the next number in the sequence.

**1.4.** If you assume that the coins are currently circulated U.S. currency, you would not be able to find a solution. If, however, you consider that the word "cents" is also used to refer to hundredths of other currencies, then all that would be required is that increments of 10 and 20 centers exist in some currency. As an example, "cents" may refer to hundredths of a Euro. You would say the coins must be worth 10 and 20 Euro cents, respectively (which do exist).

**1.5.** There are 12 ways:

| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 |
| 3 | 1 | 4 | 2 |
| 2 | 4 | 1 | 3 |

| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 |
| 2 | 1 | 4 | 3 |
| 3 | 4 | 1 | 2 |

| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 |
| 2 | 4 | 1 | 3 |
| 3 | 1 | 4 | 2 |

| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 |
| 3 | 4 | 1 | 2 |
| 2 | 1 | 4 | 3 |

| | | | |
|---|---|---|---|
| 4 | 3 | 1 | 2 |
| 1 | 2 | 3 | 4 |
| 2 | 1 | 4 | 3 |
| 3 | 4 | 2 | 1 |

| | | | |
|---|---|---|---|
| 4 | 3 | 1 | 2 |
| 1 | 2 | 3 | 4 |
| 3 | 4 | 1 | 2 |
| 2 | 1 | 4 | 3 |

| | | | |
|---|---|---|---|
| 4 | 3 | 1 | 2 |
| 1 | 2 | 4 | 3 |
| 2 | 1 | 3 | 4 |
| 3 | 4 | 2 | 1 |

| | | | |
|---|---|---|---|
| 4 | 3 | 1 | 2 |
| 1 | 2 | 4 | 3 |
| 2 | 4 | 3 | 1 |
| 3 | 1 | 2 | 4 |

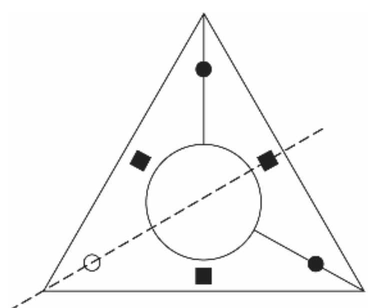
| | | | |
|---|---|---|---|
| 4 | 3 | 1 | 2 |
| 1 | 2 | 4 | 3 |
| 3 | 1 | 2 | 4 |
| 2 | 4 | 3 | 1 |

| | | | |
|---|---|---|---|
| 4 | 3 | 1 | 2 |
| 1 | 2 | 4 | 3 |
| 3 | 4 | 2 | 1 |
| 2 | 1 | 3 | 4 |

| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |
| 1 | 2 | 4 | 3 |
| 2 | 1 | 3 | 4 |
| 3 | 4 | 1 | 2 |

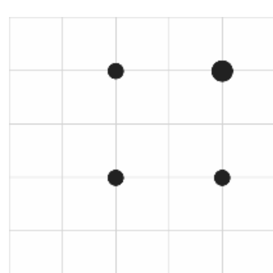
| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |
| 1 | 2 | 4 | 3 |
| 3 | 4 | 1 | 2 |
| 2 | 1 | 3 | 4 |

**1.6.** There is one axis of reflection symmetry. It is marked by the dashed line:



**1.7.** One. An axis of rotational symmetry is an axis about which the object can be rotated (through some angle other than a multiple of 360 degrees), that results in an indistinguishable appearance compared to the original orientation of the object. For a cone the axis passing through the center of the circular face and through the vertex (point) of the cone is the only axis of rotational symmetry.

**1.8.** One unit down and left from the upper right corner. This way the coins form a square:

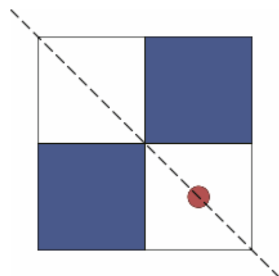


**1.9.** "T" and "A" are reflection symmetric across a vertical line passing through the center of the letter. "E" and "B" are reflection symmetric across a horizontal line passing through the center of the letter. "L" and "S" have no reflection symmetry.

**1.10.** Reflection and rotation symmetry. A sphere is reflection symmetric across any plane that passes through its center, and rotationally symmetric around any axis passing through its center.

**1.11.** A cube has 9 planes of reflection symmetry and 13 axes of rotational symmetry. 3 planes each bisect four sides of the cube. The remaining six planes pass through edges that are diagonally opposite each other. 3 of the axes of rotational symmetry pass orthogonally through the centers of two square faces. 4 of the axes pass through corners that are opposite each other along the body diagonal. 6 axes pass through the centers of edges that are opposite each other along the body diagonal.

**1.12.** There is one axis of reflection symmetry:



**1.13.** The maximum number of axes of reflection symmetry is two. There are two sides about which we have no information. Let us assume the side opposite the visible blue side is also blue, and the side opposite the visible red side is also red. In that case, the object would be reflection symmetric about a vertical plane that bisects both blue sides, and a about a vertical plane that bisects both red sides.

**1.14.** We use $v_x = \Delta x/\Delta t$ for constant speed in the $x$ direction to write:

$$\Delta x = v_x \Delta t = \frac{299{,}792{,}458\text{ m}}{1\text{s}} \times \frac{3600\text{ s}}{1\text{ h}} \times \frac{24\text{ h}}{1\text{ day}} \times \frac{365.25\text{ days}}{1\text{ yr}} \times 78\text{ yrs} = 7.4\times 10^{17}\text{ m}$$

**1.15.** (*a*) We convert units using known conversion factors:

$$9.3\times 10^{7}\text{ miles} \times \frac{1609\text{ m}}{1\text{ mi}} \times \frac{10^{3}\text{ mm}}{1\text{ m}} = 1.5\times 10^{14}\text{ mm}$$

(*b*) If we divide the distance by the width of Earth, that will tell us how many Earths can fit in that distance.

$$\text{Number of Earths} = \frac{\text{Distance to Sun}}{2R_{\text{Earth}}} = \frac{9.3\times 10^{7}\text{ mi} \times \dfrac{1.609\text{ km}}{1\text{ mi}}}{2(6.38\times 10^{3}\text{ km})} = 1.2\times 10^{4}\text{ Earths}$$

**1.16.** The blue whale and a human have densities that are the same orders of magnitude, and contain similar concentrations of different atom types. Hence it is reasonable to say that the ratio of the numbers of atoms in these two species should be roughly equal to the ratio of volumes of these two species: $\dfrac{V_{\text{bw}}}{V_{\text{human}}} = \dfrac{10^{32}}{10^{29}} = 10^{3}.$ Since whales and people are three dimensional, this corresponds to blue whales being roughly one order of magnitude larger in each of three dimensions. Hence, the blue whale is approximately 10 times longer than a human is tall.

**1.17.** Because the gastrotrich lifetime is given as three days, this should technically be treated as being on the order of one day. However, this is an approximate value of a lifetime, and if it were slightly higher, it would be treated as being on the order of ten days. Either of these is defensible in order of magnitude treatments. One tortoise lifetime can be related to gastrotrich lifetimes using the following order of magnitude conversions:

$$10^2\ \text{yr}\times\frac{10^3\ \text{days}}{1\ \text{yr}}\times\frac{1\ \text{gastrotrich lifetime}}{1\ \text{day}}=10^5\ \text{gastrotrich lifetimes}$$

If we had used 10 days as the order of magnitude for a gastrotrich lifetime we would have obtained $10^4$ gastrotrich lifetimes. Hence there are 10,000 to 100,000 gastrotrich lifetimes in one tortoise lifetime. Either of these is acceptable.

**1.18.** Given the diameter of a droplet of water, we can estimate the volume of a droplet using $V_{\text{drop}}=\frac{4}{3}\pi r^3\approx 1\times 10\times(10^{-3}\ \text{m})^3=10^{-8}\ \text{m}^3$. We can estimate the volume of a human body to be on the order of $V_{\text{body}}=10^{-1}\ \text{m}^3$. Hence an order of magnitude estimate of the number of droplets in the human body is given by $\frac{V_{\text{body}}}{V_{\text{drop}}}=10^7$.

**1.19.** Answers may vary by an order of magnitude since some textbooks may be somewhat thicker than 3.0 cm, and others may be thinner. My textbook has a thickness that is on the order of $10^{-1}$ m. The distance to the moon is $3.84\times10^8$ m, meaning the distance is of order $10^9$ m. Dividing the distance by the thickness of one textbook yields $\text{Number of books}=\frac{\text{Distance to Moon}}{\text{Thickness of book}}=\frac{10^9}{10^{-1}}=10^{10}$. Hence $10^{10}$ copies of my physics textbook could fit between Earth and the Moon.

**1.20.** We proceed by finding the total mass of water in the pool, and dividing this by the mass of a single molecule of water:

$N=\frac{m_{\text{pool}}}{m_{\text{molecule}}}=\frac{\rho_{\text{water}}V_{\text{pool}}}{M/N_{\text{A}}}$ where $\rho_{\text{water}}=1000\ \text{kg/m}^3$ is the density of water, $V_{\text{pool}}=(15\times8.5\times1.5)\ \text{m}^3$ is the volume of the swimming pool, $M=0.018\,\text{kg}$ is the molar mass of water, and $N_{\text{A}}=6.02\times10^{23}\ \text{mol}^{-1}$ is Avogadro's number. Using this information we find

$$N=\frac{(1.0\times10^3\ \text{kg/m}^3)(191\ \text{m}^3)}{(0.018\ \text{kg/mol})}(6.02\times10^{23}\ \text{mol}^{-1})=6.4\times10^{30}\ \text{molecules in the pool.}$$

**1.21.** (*a*) Since numbers are not given, it might be natural to cancel lengths and be left with a factor of $2^3$. In that case the volume of the cube would increase by one order of magnitude. But if one were asked to use an order of magnitude estimate to first express $\ell_2$ in terms of $\ell_1$, one might find that they have the same order of magnitude and that the volume therefore does not increase. Either of these answers (one order of magnitude, or zero orders of magnitude) is acceptable. (*b*) Yes, because of the rules of rounding numerical values. For example, if $V_1=3.5\ \text{m}^3$, that value would round to an order of magnitude of 10 m$^3$. Then $V_2=8V_1=28\ \text{m}^3$, which also rounds to an order of magnitude of 10 m$^3$.

**1.22.** The speed of light is of order $10^8$ m/s. The length of Earth's trip around the Sun is of order $10^{12}$ m. Hence the order of magnitude of the time light would need to make the same trip around the sun is $\text{time}=\frac{\text{distance}}{\text{speed}}=\frac{10^{12}\ \text{m}}{10^8\ \text{m/s}}=10^4\ \text{s}$. Hence light would take approximately $10^4$ s to complete this trip.

**1.23.** The surface area of the roughly spherical distribution of leaves is $4\pi r^2$, where $r$ is the radius of the tree's sphere of leaves. The surface area of an individual leaf is just its length $\ell$ times its width $w$. We can find the number of leaves by dividing the total surface area by the area of one leaf: $N=\frac{4\pi r^2}{\ell w}=\frac{10(10\ \text{m})^2}{(0.1\text{m})(0.1\ \text{m})}=10^5$ leaves.

**1.24.** To find the number of generations since the universe began, we divide the age of the universe by the time required for one human generation. We simply have to get both times in the same units. Let us express one human generation in seconds: $30\ \text{yr} \times \frac{365.25\ \text{days}}{1\,\text{yr}} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\text{h}} = 9.5\times 10^{8}\ \text{s}.$ Hence the number of human generations $N$ is given by $N = \frac{t_{\text{universe}}}{t_{\text{generation}}} = \frac{10^{17}\ \text{s}}{9.5\times 10^{8}\ \text{s}} = 10^{8}$ generations.

**1.25.** Not reasonable. Because light travels much faster than sound, any thunder peal is delayed compared to the light signal caused by the lightning bolt event. From the principle of causality, the lightning you see after you hear the peal cannot have caused the peal. The peal you heard must have come from a previous lightning strike.

**1.26.** Call the period of time required for one such oscillation *T*. Then the second is defined such that $1.0\ \text{s} = (9.19\times 10^{9})T$ or $T = \frac{1.0\ \text{s}}{(9.19\times 10^{9})} = 1.09\times 10^{-10}\ \text{s}.$

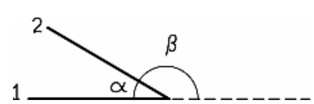
**1.27.** That the barrier lowers time after time 30 s before a train passes is consistent with a causal relationship between the two events. The single negative result, however, tells you that the lowering of the barrier cannot be the *direct* cause of the passing of the train. More likely, the lowering is triggered when the train passes a sensor quite a distance up the tracks from the barrier and the sensor sends an electrical signal to the lowering mechanism. A malfunction in either the sensor, the electrical connections, or the lowering mechanism would account for the one negative result you observed.

**1.28.** The period calculated in Problem 26 was $T = 1.09\times 10^{-10}\ \text{s}.$ The speed of light is $c = 3.00\times 10^{8}\ \text{m/s}.$ Here the distance travelled is just the speed times the time: $d = c\Delta t = (3.00\times 10^{8}\ \text{m/s})(1.09\times 10^{-10}\ \text{s}) = 0.0326\ \text{m}$.

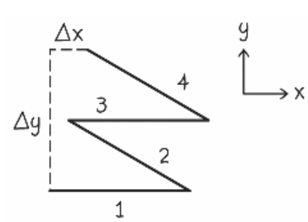
**1.29.** $E = mc^2$ Here, $E$ is the type of energy described, $m$ is the mass of the object, and $c$ is the speed of light.

**1.30.** Rectangle, parallelogram, and two equilateral triangles meeting at a vertex forming an hourglass shape.

**1.31.** The problem states that two adjacent sides must make an angle of $30°$. This most likely means the interior angle between actual sides. This angle is labeled $\alpha$ in the figure below. But one might also describe the exterior angle in this way. This angle is labeled $\beta$ below. If one accepts this interpretation, some of the $30°$ angles could be interior and some could be exterior.



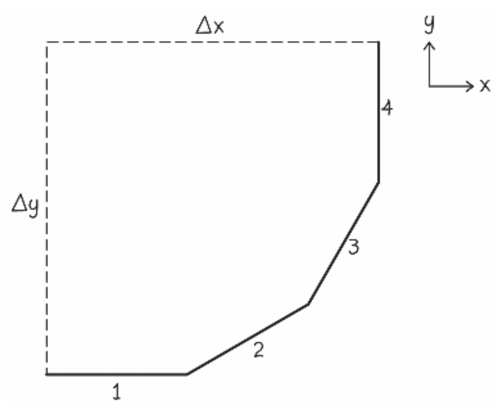

Let us first address the case in which the $30°$ angle refers to the interior angle. Remembering that the segments cannot cross, there is only one possible arrangement of segments that fits the description:

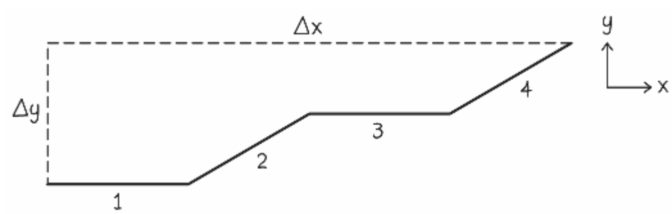


Clearly $\Delta y = 2\ell\sin(\theta)$ and $\Delta x = 2\ell(1-\cos(\theta))$. The Pythagorean Theorem tells us that the distance between the two unconnected points must be $d = \sqrt{\Delta x^2 + \Delta y^2} = 2\ell\left((1-\cos(30°))^2 + \sin^2(30°)\right)^{1/2} = 1.0\ell.$

So the distance between unconnected ends is $\ell$.

If the $30°$ angle refers to the exterior angle, we can obtain several possible shapes. The two shapes that give the shortest and longest distances are shown below:



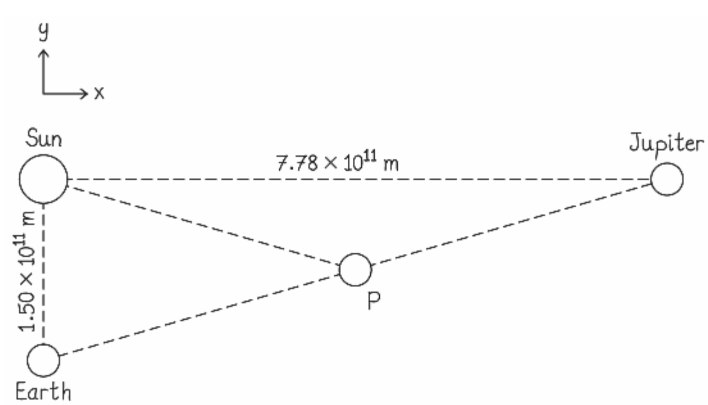

We can see in the top figure (that yields the shortest distance) $\Delta x = \ell(1+\cos(30°)+\cos(60°)+0)$, and since the $x$ and $y$ distances are equal based on symmetry, we find $d = \sqrt{2}\ell(1+\cos(30°)+\cos(60°)) = 3.3\ell.$

In the second figure (that yields the greatest distance) we find $\Delta x = 2\ell(1+\cos(30°))$ and $\Delta x = 2\ell(\sin(30°))$. Again using the Pythagorean Theorem yields a distance of $d = \left[(2\ell(1+\cos(30°)))^2 + (2\ell\sin(30°))^2\right]^{1/2} = 3.9\ell.$

If a combination of interior and exterior angles is used, there are even more possibilities. The shortest of these distances is 0 (parallelogram). It can be shown that other possible distances include $2.0\ell,\ 2.2\ell,\ 2.4\ell.$

**1.32.** Uncle, cousin, grandmother, aunt, grandfather, brother.

**1.33.** Consider the diagram below.



The distance from the Sun to point P is

$$\sqrt{\Delta x^2+\Delta y^2}=(1/2)\sqrt{(1.50\times10^{11}\text{ m})^2+(7.78\times10^{11}\text{ m})^2}=3.96\times10^{11}\text{ m}$$

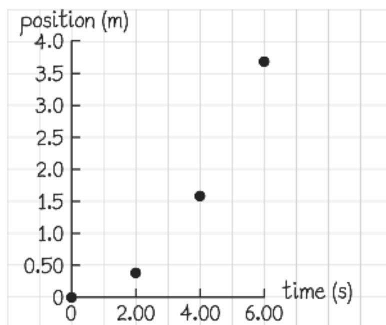
The time that light would require to cross this distance can be found using

$$\Delta t=d/v=(3.96\times10^{11}\text{ m})/(3.00\times10^{8}\text{ m/s})=1.32\times10^{3}\text{ s}$$

which is equivalent to about 22.0 minutes.

**1.34.**

| Time (s) | Position (m) |
|---|---|
| 0.00 | 0.00 |
| 2.00 | 0.40 |
| 4.00 | 1.59 |
| 6.00 | 3.64 |




**1.35.** (*a*) The position decreases linearly as a function of time, from an initial position of 4.0 m to a final position of zero at a time of 8.0 s, with a slope of $-0.5$ m/s. (*b*) $x(t)=mt+b$ where $m=-0.5$ m/s, and $b=4.0$ m.

**1.36.** Forming a regular tetrahedron from these triangles will automatically satisfy all conditions.

**1.37.** $8.95\text{ m}\times\dfrac{1\text{ ft}}{0.3048\text{ m}}\times\dfrac{12\text{ inch}}{1\text{ ft}}=352\text{ inches}$

**1.38.** We change units using known conversion factors:

$$35{,}000\text{ ft}\times\frac{1\text{ mile}}{5280\text{ ft}}=6.629\text{ miles}$$

$$35{,}000\text{ ft}\times\frac{12\text{ in}}{1\text{ ft}}\times\frac{25.4\text{ mm}}{1\text{ in}}\times\frac{1\text{ m}}{10^3\text{ mm}}\times\frac{1\text{ km}}{10^3\text{ m}}=10.7\text{ km}$$

**1.39.** (*a*) The density will be the same. (*b*) The density will be the same.

**1.40.** We change units using known conversion factors:

$$\frac{2.9979\times10^8\text{ m}}{1\text{ s}}\times\frac{1\text{ mile}}{1609\text{ m}}=1.863\times10^5\text{ miles/s}$$

$$\frac{2.9979\times10^8\text{ m}}{1\text{ s}}\times\frac{1\text{ s}}{10^9\text{ ns}}\times\frac{10^3\text{ mm}}{1\text{m}}\times\frac{1\text{ in}}{25.4\text{ mm}}=11.8\text{ inches/ns}$$

$$\frac{2.9979\times10^8\ \text{m}}{1\ \text{s}}\times\frac{1\ \text{km}}{1000\ \text{m}}\times\frac{3600\ \text{s}}{1\ \text{hr}}=1.079\times10^9\ \text{km/h}$$

**1.41.** If the two stones are made from the same material, they should have roughly the same density. We calculate the density of each stone and compare them:

$$\rho_1=\frac{m_1}{V_1}=\frac{2.9\times10^{-2}\ \text{kg}}{10.0\ \text{cm}^3}=2.9\times10^{-3}\ \text{kg/cm}^3$$

$$\rho_2=\frac{m_2}{V_2}=\frac{2.5\times10^{-2}\ \text{kg}}{7.50\ \text{cm}^3}=3.3\times10^{-3}\ \text{kg/cm}^3$$

No, it is not likely. Stone 2 has considerably higher density.

**1.42.** This length can be expressed as $1.000\ \text{mi}+440\ \text{yd}\times\frac{3\ \text{ft}}{1\ \text{yd}}\times\frac{1\ \text{mi}}{5{,}280\ \text{ft}}=1.250\ \text{mi}.$ We now convert this entirely to feet: $1.250\ \text{mi}\times\frac{5280\ \text{ft}}{1\ \text{mi}}=6{,}600\ \text{ft}.$

**1.43.** We can find the units of $A$ from given information: $\text{units}[A]=\text{m/s}^2\cdot\text{s}^2=\text{m}.$ If $A$ has units of meters, and we wish to add $A$ and $B$, then $B$ must also have units of meters.

**1.44.** Mass density is the ratio of mass per unit volume. While one could have any mass of a given substance, or any volume of that substance, the density tends to be a constant value for a given material (under certain conditions).

**1.45.** For all cases we find the order of magnitude of the mass of Earth using $m=\rho V=\rho\left(\frac{4}{3}\pi R_\text{E}^3\right).$

(*a*) $m=\rho_\text{air}\left(\frac{4}{3}\pi R_\text{E}^3\right)=(10^0\ \text{kg/m}^3)(1\times10\times(10^7\ \text{m})^3)=10^{22}\ \text{kg}$

(*b*) $m=\rho_{5515}\left(\frac{4}{3}\pi R_\text{E}^3\right)=(10^4\ \text{kg/m}^3)(1\times10\times(10^7\ \text{m})^3)=10^{26}\ \text{kg}$

(*c*) $m=\rho_\text{nucleus}\left(\frac{4}{3}\pi R_\text{E}^3\right)=(10^{18}\ \text{kg/m}^3)(1\times10\times(10^7\ \text{m})^3)=10^{40}\ \text{kg}$

**1.46.** We rearrange the given expression to solve for $y$ and then write all units in terms of SI base units and powers of ten:

$$y=\left(\frac{x}{a}\right)^{2/3}=\left(\frac{61.7\ \text{Eg}\cdot\text{fm}^2/\text{ms}^3}{7.81\ \mu\text{g/Tm}}\right)^{2/3}$$

$$y=\left(\frac{61.7\ (10^{18}\ \text{g})\cdot(10^{-15}\ \text{m})^2/(10^{-3}\ \text{s})^3}{7.81\ (10^{-6}\ \text{g})/(10^{12}\ \text{m})}\right)^{2/3}$$

$$y=3.97\times10^{10}\ \text{m}^2/\text{s}^2$$

**1.47.** (*a*) $3.00\times10^8\ \text{m/s}$ (*b*) $8.99\times10^{16}\ \text{m}^2/\text{s}^2$ (*c*) No. There is a small difference because the answer to (*a*) was rounded before squaring. The answer to (*b*) was obtained using more digits of the speed of light, and only the result was rounded to three significant digits.

**1.48.** The given distance can also be written as 1.25 miles. We now convert to kilometers using known the known conversion factor:

$$1.25\ \text{mi}\times\frac{1.609\ \text{km}}{1\ \text{mi}}=2.012\ \text{km}$$

**1.49.** Your answer has four significant digits. When dividing a quantity by an integer, the number of significant digits should not change.

**1.50.** Yes, there is a difference in the precision. You will calculate your gas mileage by dividing the number of miles you travel by the gallons of gasoline used. Since the gas pump gives you thousandths (and most vehicles take 10 gal or more) you know the fuel used to five significant digits. Neither distance given has this many, making the precision of the distance value the limiting factor. Thus you can calculate mileage to three significant digits when you use 40.0 mi for distance and to four significant digits when you use 400.0 mi.

**1.51.** The odometer should still say 35,987.1 km. 47.00 m is only 0.04700 km. Because the odometer reading is only given to the nearest tenth of a kilometer, the sum (final odometer reading) must also be given only to the tenth of a kilometer.

**1.52.** We convert the given amount of ingested caffeine using known conversion factors:

$$\frac{34\text{ mg}}{\text{serving}}\times\frac{2\text{ servings}}{\text{day}}\times\frac{365.25\text{ days}}{1\text{ yr}}\times\frac{1\text{g}}{10^3\text{ mg}}\times\frac{1\text{ mol}}{194.19\text{ g}}\times\frac{6.02\times10^{23}\text{ molecules}}{1\text{mol}}=7.7\times10^{22}\text{ molecules/yr}$$

**1.53.** The molarity of the solution must be known to a precision of 1 part in 15. Since we can measure volume to arbitrary precision and we have the molar mass to very high precision, the limiting factor on the precision of our molarity is the precision of the mass we use. Hence the precision of the mass we use must be at least as good as the required precision of the molarity. We can only measure tenths of a gram, so we must have at least 1.5 grams in order to know the precision of our mass to 1 part in 15 or better. Mathematically:

$$\frac{m}{MV}\pm\frac{\Delta m}{MV}=(0.15\pm0.01)\text{ mol/L}$$

$$0.1\text{ g }\le\Delta m=0.01MV$$

$$V\ge\frac{0.1\text{ g}}{(0.01\text{ mol/L})(58.44\text{ g/mol})}$$

$$V\ge0.17\text{ L}$$

$$m=0.15(MV)=1.5\text{ g}$$

As can be seen from the above calculation, the answer could also be given in terms of the minimum volume of solution prepared. That minimum volume is 0.17 L.

**1.54.** The mass density is given by $\rho=m/V$. The volume is given in milliliters, which are equivalent to cubic centimeters. Hence

$$\rho=\frac{25.403\text{ g}}{23.42\text{ cm}^3}\times\frac{10^{-3}\text{ kg}}{1\text{ g}}\times\frac{10^6\text{ cm}^3}{1\text{ m}^3}=1.085\times10^3\text{ kg/m}^3$$

**1.55.** The mass evaporated each second is given by

$$\begin{aligned}\frac{|\Delta m|}{\Delta t}&=\frac{m_{\text{dish}}+m_{\text{liquid}}-m_{\text{final}}}{\Delta t}\\&=\frac{(145.67\text{ g})+(0.335\text{ g})-(145.82\text{ g})}{(25.01\text{ s})}\\&=7.4\times10^{-3}\text{ g/s}\end{aligned}$$

Note that we have only two significant digits. Because two of the masses were only known to the hundredths place, we only had two digits of precision after taking the sum (and difference) of masses.

**1.56.** There are too many significant digits. The volume is only given to two significant digits. The answer could have at most two significant digits.