# Chapter 1
# Systems of Measurement

## Conceptual Problems

***1 •**

**Determine the Concept** The fundamental physical quantities in the SI system include mass, length, and time. Force, being the product of mass and acceleration, is not a fundamental quantity. $\boxed{(c)\text{ is correct.}}$

**2 •**

**Picture the Problem** We can express and simplify the ratio of m/s to m/s$^2$ to determine the final units.

Express and simplify the ratio of m/s to m/s$^2$:

$$\frac{\frac{\text{m}}{\text{s}}}{\frac{\text{m}}{\text{s}^2}} = \frac{\text{m}\cdot\text{s}^2}{\text{m}\cdot\text{s}} = \text{s and } \boxed{(d)\text{ is correct.}}$$

**3 •**

**Determine the Concept** Consulting Table 1-1 we note that the prefix giga means $10^9$. $\boxed{(c)\text{ is correct.}}$

**4 •**

**Determine the Concept** Consulting Table 1-1 we note that the prefix mega means $10^6$. $\boxed{(d)\text{ is correct.}}$

***5 •**

**Determine the Concept** Consulting Table 1-1 we note that the prefix pico means $10^{-12}$. $\boxed{(a)\text{ is correct.}}$

**6 •**

**Determine the Concept** Counting from left to right and ignoring zeros to the left of the first nonzero digit, the last significant figure is the first digit that is in doubt. Applying this criterion, the three zeros after the decimal point are not significant figures, but the last zero is significant. Hence, there are four significant figures in this number. $\boxed{(c)\text{ is correct.}}$

**7** •

**Determine the Concept** Counting from left to right, the last significant figure is the first digit that is in doubt. Applying this criterion, there are six significant figures in this number. $\boxed{(e)\text{ is correct.}}$

**8** •

**Determine the Concept** The advantage is that the length measure is always with you. The disadvantage is that arm lengths are not uniform; if you wish to purchase a board of ″two arm lengths″ it may be longer or shorter than you wish, or else you may have to physically go to the lumberyard to use your own arm as a measure of length.

**9** •

(*a*) True. You cannot add ″apples to oranges″ or a length (distance traveled) to a volume (liters of milk).

(*b*) False. The distance traveled is the product of speed (length/time) multiplied by the time of travel (time).

(*c*) True. Multiplying by any conversion factor is equivalent to multiplying by 1. Doing so does not change the value of a quantity; it changes its units.

## Estimation and Approximation

***10** ••

**Picture the Problem** Because $\theta$ is small, we can approximate it by $\theta \approx D/r_m$ provided that it is in radian measure. We can solve this relationship for the diameter of the moon.

Express the moon's diameter $D$ in terms of the angle it subtends at the earth $\theta$ and the earth-moon distance $r_m$:

$$D = \theta r_m$$

Find $\theta$ in radians:

$$\theta = 0.524° \times \frac{2\pi\,\text{rad}}{360°} = 0.00915\,\text{rad}$$

Substitute and evaluate $D$:

$$D = (0.00915\,\text{rad})(384\,\text{Mm}) = \boxed{3.51 \times 10^6\,\text{m}}$$

***11 ••**

**Picture the Problem** We'll assume that the sun is made up entirely of hydrogen. Then we can relate the mass of the sun to the number of hydrogen atoms and the mass of each.

Express the mass of the sun $M_S$ as the product of the number of hydrogen atoms $N_H$ and the mass of each atom $M_H$:

$$M_S = N_H M_H$$

Solve for $N_H$:

$$N_H = \frac{M_S}{M_H}$$

Substitute numerical values and evaluate $N_H$:

$$N_H = \frac{1.99\times10^{30}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = \boxed{1.19\times10^{57}}$$

**12 ••**

**Picture the Problem** Let $P$ represent the population of the United States, $r$ the rate of consumption and $N$ the number of aluminum cans used annually. The population of the United States is roughly $3\times10^8$ people. Let's assume that, on average, each person drinks one can of soft drink every day. The mass of a soft-drink can is approximately $1.8\times10^{-2}$ kg.

(*a*) Express the number of cans $N$ used annually in terms of the daily rate of consumption of soft drinks $r$ and the population $P$:

$$N = rP\Delta t$$

Substitute numerical values and approximate $N$:

$$N = \left(\frac{1\,\text{can}}{\text{person}\cdot\text{d}}\right)\left(3\times10^8\text{ people}\right)\times(1\text{ y})\left(365.24\frac{\text{d}}{\text{y}}\right)$$

$$\approx \boxed{10^{11}\text{ cans}}$$

(*b*) Express the total mass of aluminum used per year for soft drink cans $M$ as a function of the number of cans consumed and the mass $m$ per can:

$$M = Nm$$

Substitute numerical values and evaluate $M$:

$$M = \left(10^{11}\ \text{cans/y}\right)\left(1.8\times10^{-2}\ \text{kg/can}\right) \approx \boxed{2\times10^{9}\ \text{kg/y}}$$

(*c*) Express the value of the aluminum as the product of $M$ and the value at recycling centers:

$$\begin{aligned}\text{Value} &= (\$1/\text{kg})M\\ &= (\$1/\text{kg})\left(2\times10^{9}\ \text{kg/y}\right)\\ &= \$2\times10^{9}/\text{y}\\ &= \boxed{2\ \text{billion dollars/y}}\end{aligned}$$

**13** ••

**Picture the Problem** We can estimate the number of words in *Encyclopedia Britannica* by counting the number of volumes, estimating the average number of pages per volume, estimating the number of words per page, and finding the product of these measurements and estimates. Doing so in *Encyclopedia Britannica* leads to an estimate of approximately 200 million for the number of words. If we assume an average word length of five letters, then our estimate of the number of letters in *Encyclopedia Britannica* becomes $10^9$.

(*a*) Relate the area available for one letter $s^2$ and the number of letters $N$ to be written on the pinhead to the area of the pinhead:

$$Ns^2 = \frac{\pi}{4}d^2$$ where $d$ is the diameter of the pinhead.

Solve for $s$ to obtain:

$$s = \sqrt{\frac{\pi d^2}{4N}}$$

Substitute numerical values and evaluate $s$:

$$s = \sqrt{\frac{\pi\left[\left(\tfrac{1}{16}\ \text{in}\right)\left(2.54\ \frac{\text{cm}}{\text{in}}\right)\right]^2}{4\left(10^9\right)}} \approx \boxed{10^{-8}\ \text{m}}$$

(*b*) Express the number of atoms per letter $n$ in terms of $s$ and the atomic spacing in a metal $d_{\text{atomic}}$:

$$n = \frac{s}{d_{\text{atomic}}}$$

Substitute numerical values and evaluate $n$:

$$n = \frac{10^{-8}\ \text{m}}{5\times10^{-10}\ \text{atoms/m}} \approx \boxed{20\ \text{atoms}}$$

***14** ••

**Picture the Problem** The population of the United States is roughly $3 \times 10^8$ people. Assuming that the average family has four people, with an average of two cars per family, there are about $1.5 \times 10^8$ cars in the United States. If we double that number to include trucks, cabs, etc., we have $3 \times 10^8$ vehicles. Let's assume that each vehicle uses, on average, about 12 gallons of gasoline per week.

(*a*) Find the daily consumption of gasoline *G*:

$$G = (3\times10^8 \text{ vehicles})(2\,\text{gal/d}) = 6\times10^8 \text{ gal/d}$$

Assuming a price per gallon $P = \$1.50$, find the daily cost *C* of gasoline:

$$C = GP = (6\times10^8 \text{ gal/d})(\$1.50/\text{gal}) = \$9\times10^8/\text{d} \approx \boxed{\$1\,\text{billion dollars/d}}$$

(*b*) Relate the number of barrels *N* of crude oil required annually to the yearly consumption of gasoline *Y* and the number of gallons of gasoline *n* that can be made from one barrel of crude oil:

$$N = \frac{Y}{n} = \frac{G\Delta t}{n}$$

Substitute numerical values and estimate *N*:

$$N = \frac{(6\times10^8 \text{ gal/d})(365.24\,\text{d/y})}{19.4\,\text{gal/barrel}} \approx \boxed{10^{10} \text{ barrels/y}}$$

**15** ••

**Picture the Problem** We'll assume a population of 300 million (fairly accurate as of September, 2002) and a life expectancy of 76 y. We'll also assume that a diaper has a volume of about half a liter. In (*c*) we'll assume the disposal site is a rectangular hole in the ground and use the formula for the volume of such an opening to estimate the surface area required.

(*a*) Express the total number *N* of disposable diapers used in the United States per year in terms of the number of children *n* in diapers and the number of diapers *D* used by each child in 2.5 y:

$$N = nD$$

Use the daily consumption, the number of days in a year, and the estimated length of time a child is in diapers to estimate the number of diapers *D* required per child:

$$D = \frac{3\,\text{diapers}}{\text{d}} \times \frac{365.24\,\text{d}}{\text{y}} \times 2.5\,\text{y} \approx 3\times10^3 \text{ diapers/child}$$

Use the assumed life expectancy to estimate the number of children $n$ in diapers:

$$n=\left(\frac{2.5\,\text{y}}{76\,\text{y}}\right)\left(300\times10^{6}\ \text{children}\right)$$
$$\approx 10^{7}\ \text{children}$$

Substitute to obtain:

$$N=\left(10^{7}\ \text{children}\right)\times\left(3\times10^{3}\ \text{diapers/child}\right)$$
$$\approx \boxed{3\times10^{10}\ \text{diapers}}$$

(*b*) Express the required landfill volume $V$ in terms of the volume of diapers to be buried:

$$V=NV_{\text{one diaper}}$$

Substitute numerical values and evaluate $V$:

$$V=\left(3\times10^{10}\ \text{diapers}\right)\left(0.5\,\text{L/diaper}\right)$$
$$\approx \boxed{1.5\times10^{7}\ \text{m}^{3}}$$

(*c*) Express the required volume in terms of the volume of a rectangular parallelepiped:

$$V=Ah$$

Solve and evaluate $h$:

$$A=\frac{V}{h}=\frac{1.5\times10^{7}\ \text{m}^{3}}{10\,\text{m}}=1.5\times10^{6}\ \text{m}^{2}$$

Use a conversion factor to express this area in square miles:

$$A=1.5\times10^{6}\ \text{m}^{2}\times\frac{1\ \text{mi}^{2}}{2.590\,\text{km}^{2}}$$
$$\approx \boxed{0.6\,\text{mi}^{2}}$$

**16** •••

**Picture the Problem** The number of bits that can be stored on the disk can be found from the product of the capacity of the disk and the number of bits per byte. In part (*b*) we'll need to estimate (*i*) the number of bits required for the alphabet, (*ii*) the average number of letters per word, (*iii*) an average number of words per line, (*iv*) an average number of lines per page, and (*v*) a book length in pages.

(*a*) Express the number of bits $N_{\text{bits}}$ as a function of the number of bits per byte and the capacity of the hard disk $N_{\text{bytes}}$:

$$N_{\text{bits}}=N_{\text{bytes}}\left(8\,\text{bits/byte}\right)$$
$$=\left(2\times10^{9}\ \text{bytes}\right)\left(8\,\text{bits/byte}\right)$$
$$=\boxed{1.60\times10^{10}\ \text{bits}}$$

(*b*) Assume an average of 8 letters/word and 8 bits/character to estimate the number of bytes required per word:

$$8\,\frac{\text{bits}}{\text{character}}\times 8\,\frac{\text{characters}}{\text{word}} = 64\,\frac{\text{bits}}{\text{word}} = 8\,\frac{\text{bytes}}{\text{word}}$$

Assume 10 words/line and 60 lines/page:

$$600\,\frac{\text{words}}{\text{page}}\times 8\,\frac{\text{bytes}}{\text{word}} = 4800\,\frac{\text{bytes}}{\text{page}}$$

Assume a book length of 300 pages and approximate the number bytes required:

$$300\,\text{pages}\times 4800\,\frac{\text{bytes}}{\text{page}} = 1.44\times 10^{6}\,\text{bytes}$$

Divide the number of bytes per disk by our estimated number of bytes required per book to obtain an estimate of the number of books the 2-gigabyte hard disk can hold:

$$N_{\text{books}} = \frac{2\times 10^{9}\,\text{bytes}}{1.44\times 10^{6}\,\text{bytes/book}} \approx \boxed{1400\,\text{books}}$$

***17** ••

**Picture the Problem** Assume that, on average, four cars go through each toll station per minute. Let *R* represent the yearly revenue from the tolls. We can estimate the yearly revenue from the number of lanes *N*, the number of cars per minute *n*, and the $6 toll per car *C*.

$$R = NnC = 14\,\text{lanes}\times 4\,\frac{\text{cars}}{\text{min}}\times 60\,\frac{\text{min}}{\text{h}}\times 24\,\frac{\text{h}}{\text{d}}\times 365.24\,\frac{\text{d}}{\text{y}}\times\frac{\$6}{\text{car}} = \boxed{\$177\text{M}}$$

## Units

**18** •

**Picture the Problem** We can use the metric prefixes listed in Table 1-1 and the abbreviations on page EP-1 to express each of these quantities.

(*a*)

$$1{,}000{,}000\,\text{watts} = 10^{6}\,\text{watts} = \boxed{1\,\text{MW}}$$

(*b*)

$$0.002\,\text{gram} = 2\times 10^{-3}\,\text{g} = \boxed{2\,\text{mg}}$$

(*c*)

$$3\times 10^{-6}\,\text{meter} = \boxed{3\,\mu\text{m}}$$

(*d*)

$$30{,}000\,\text{seconds} = 30\times 10^{3}\,\text{s} = \boxed{30\,\text{ks}}$$

**19** •

**Picture the Problem** We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without prefixes.

(*a*)

$$40\,\mu\text{W} = 40\times10^{-6}\text{ W} = \boxed{0.000040\text{ W}}$$

(*b*)

$$4\text{ ns} = 4\times10^{-9}\text{ s} = \boxed{0.000000004\text{ s}}$$

(*c*)

$$3\text{ MW} = 3\times10^{6}\text{ W} = \boxed{3{,}000{,}000\text{ W}}$$

(*d*)

$$25\text{ km} = 25\times10^{3}\text{ m} = \boxed{25{,}000\text{ m}}$$

***20** •

**Picture the Problem** We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without abbreviations.

(*a*) $10^{-12}\text{ boo} = \boxed{1\text{picoboo}}$

(*b*) $10^{9}\text{ low} = \boxed{1\text{gigalow}}$

(*c*) $10^{-6}\text{ phone} = \boxed{1\text{microphone}}$

(*d*) $10^{-18}\text{ boy} = \boxed{1\text{attoboy}}$

(*e*) $10^{6}\text{ phone} = \boxed{1\text{megaphone}}$

(*f*) $10^{-9}\text{ goat} = \boxed{1\text{nanogoat}}$

(*g*) $10^{12}\text{ bull} = \boxed{1\text{terabull}}$

**21** ••

**Picture the Problem** We can determine the SI units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(*a*) Because $x$ is in meters, $C_1$ and $C_2t$ must be in meters:

$$\boxed{C_1 \text{ is in m; } C_2 \text{ is in m/s}}$$

(*b*) Because $x$ is in meters, ½$C_1t^2$ must be in meters:

$$\boxed{C_1 \text{ is in m/s}^2}$$

(*c*) Because $v^2$ is in m²/s², $2C_1x$ must be in $\text{m}^2/\text{s}^2$:

$$\boxed{C_1 \text{ is in m/s}^2}$$

(*d*) The argument of trigonometric function must be dimensionless; i.e. without units. Therefore, because $x$

$$\boxed{C_1 \text{ is in m; } C_2 \text{ is in s}^{-1}}$$

is in meters:

(*e*) The argument of an exponential function must be dimensionless; i.e. without units. Therefore, because $v$ is in m/s:

$\boxed{C_1 \text{ is in m/s; } C_2 \text{ is in s}^{-1}}$

**22** ••

**Picture the Problem** We can determine the US customary units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(*a*) Because $x$ is in feet, $C_1$ and $C_2t$ must be in feet:

$\boxed{C_1 \text{ is in ft; } C_2 \text{ is in ft/s}}$

(*b*) Because $x$ is in feet, $\frac{1}{2}C_1t^2$ must be in feet:

$\boxed{C_1 \text{ is in ft/s}^2}$

(*c*) Because $v^2$ is in ft$^2$/s$^2$, $2C_1x$ must be in ft$^2$/s$^2$:

$\boxed{C_1 \text{ is in ft/s}^2}$

(*d*) The argument of trigonometric function must be dimensionless; i.e. without units. Therefore, because $x$ is in feet:

$\boxed{C_1 \text{ is in ft; } C_2 \text{ is in s}^{-1}}$

(*e*) The argument of an exponential function must be dimensionless; i.e. without units. Therefore, because $v$ is in ft/s:

$\boxed{C_1 \text{ is in ft/s; } C_2 \text{ is in s}^{-1}}$

## Conversion of Units

**23** •

**Picture the Problem** We can use the formula for the circumference of a circle to find the radius of the earth and the conversion factor 1 mi = 1.61 km to convert distances in meters into distances in miles.

(*a*) The Pole-Equator distance is one-fourth of the circumference:

$c = \boxed{4 \times 10^7 \text{ m}}$

(*b*) Use the formula for the circumference of a circle to obtain:

$$R = \frac{c}{2\pi} = \frac{4\times10^{-7}\ \text{m}}{2\pi} = \boxed{6.37\times10^{6}\ \text{m}}$$

(*c*) Use the conversion factors 1 km = 1000 m and 1 mi = 1.61 km:

$$c = 4\times10^{7}\ \text{m}\times\frac{1\,\text{km}}{10^{3}\ \text{m}}\times\frac{1\,\text{mi}}{1.61\,\text{km}}$$

$$= \boxed{2.48\times10^{4}\ \text{mi}}$$

and

$$R = 6.37\times10^{6}\ \text{m}\times\frac{1\,\text{km}}{10^{3}\ \text{m}}\times\frac{1\,\text{mi}}{1.61\,\text{km}}$$

$$= \boxed{3.96\times10^{3}\ \text{mi}}$$

**24** •

**Picture the Problem** We can use the conversion factor 1 mi = 1.61 km to convert speeds in km/h into mi/h.

Find the speed of the plane in km/s:

$$v = 2(340\,\text{m/s}) = 680\,\text{m/s}$$

$$= \left(680\frac{\text{m}}{\text{s}}\right)\left(\frac{1\,\text{km}}{10^{3}\ \text{m}}\right)\left(3600\frac{\text{s}}{\text{h}}\right)$$

$$= \boxed{2450\,\text{km/h}}$$

Convert $v$ into mi/h:

$$v = \left(2450\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{mi}}{1.61\,\text{km}}\right)$$

$$= \boxed{1520\,\text{mi/h}}$$

***25** •

**Picture the Problem** We'll first express his height in inches and then use the conversion factor 1 in = 2.54 cm.

Express the player's height into inches:

$$h = 6\,\text{ft}\times\frac{12\,\text{in}}{\text{ft}} + 10.5\,\text{in} = 82.5\,\text{in}$$

Convert $h$ into cm:

$$h = 82.5\,\text{in}\times\frac{2.54\,\text{cm}}{\text{in}} = \boxed{210\,\text{cm}}$$

**26** •

**Picture the Problem** We can use the conversion factors 1 mi = 1.61 km, 1 in = 2.54 cm, and 1 m = 1.094 yd to complete these conversions.

(*a*)

$$100\,\frac{\text{km}}{\text{h}} = 100\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{mi}}{1.61\,\text{km}} = \boxed{62.1\,\frac{\text{mi}}{\text{h}}}$$

(*b*)

$$60\,\text{cm} = 60\,\text{cm} \times \frac{1\,\text{in}}{2.54\,\text{cm}} = \boxed{23.6\,\text{in}}$$

(*c*)

$$100\,\text{yd} = 100\,\text{yd} \times \frac{1\,\text{m}}{1.094\,\text{yd}} = \boxed{91.4\,\text{m}}$$

**27** •

**Picture the Problem** We can use the conversion factor 1.609 km = 5280 ft to convert the length of the main span of the Golden Gate Bridge into kilometers.

Convert 4200 ft into km:

$$4200\,\text{ft} = 4200\,\text{ft} \times \frac{1.609\,\text{km}}{5280\,\text{ft}} = \boxed{1.28\,\text{km}}$$

***28** •

**Picture the Problem** Let *v* be the speed of an object in mi/h. We can use the conversion factor 1 mi = 1.61 km to convert this speed to km/h.

Multiply *v* mi/h by 1.61 km/mi to convert *v* to km/h:

$$v\,\frac{\text{mi}}{\text{h}} = v\,\frac{\text{mi}}{\text{h}} \times \frac{1.61\,\text{km}}{\text{mi}} = \boxed{1.61v\,\text{km/h}}$$

**29** •

**Picture the Problem** Use the conversion factors 1 h = 3600 s, 1.609 km = 1 mi, and 1 mi = 5280 ft to make these conversions.

(*a*) $$1.296\times10^5\,\frac{\text{km}}{\text{h}^2} = \left(1.296\times10^5\,\frac{\text{km}}{\text{h}^2}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{36.0\,\frac{\text{km}}{\text{h}\cdot\text{s}}}$$

(*b*) $$1.296\times10^5\,\frac{\text{km}}{\text{h}^2} = \left(1.296\times10^5\,\frac{\text{km}}{\text{h}^2}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)^2\left(\frac{10^3\,\text{m}}{\text{km}}\right) = \boxed{10.0\,\frac{\text{m}}{\text{s}^2}}$$

(*c*) $$60\,\frac{\text{mi}}{\text{h}} = \left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{88.0\,\frac{\text{ft}}{\text{s}}}$$

(*d*) $$60\,\frac{\text{mi}}{\text{h}} = \left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{1.609\,\text{km}}{1\,\text{mi}}\right)\left(\frac{10^3\,\text{m}}{\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{26.8\,\frac{\text{m}}{\text{s}}}$$

**30** •

**Picture the Problem** We can use the conversion factor 1 L = 1.057 qt to convert gallons into liters and then use this gallons-to-liters conversion factor to convert barrels into cubic meters.

$$(a)\ 1\,\text{gal} = (1\,\text{gal})\left(\frac{4\,\text{qt}}{\text{gal}}\right)\left(\frac{1\,\text{L}}{1.057\,\text{qt}}\right) = \boxed{3.784\,\text{L}}$$

$$(b)\ 1\,\text{barrel} = (1\,\text{barrel})\left(\frac{42\,\text{gal}}{\text{barrel}}\right)\left(\frac{3.784\,\text{L}}{\text{gal}}\right)\left(\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{0.1589\,\text{m}^3}$$

**31** •

**Picture the Problem** We can use the conversion factor given in the problem statement and the fact that 1 mi = 1.609 km to express the number of square meters in one acre.

Multiply by 1 twice, properly chosen, to convert one acre into square miles, and then into square meters:

$$1\,\text{acre} = (1\,\text{acre})\left(\frac{1\,\text{mi}^2}{640\,\text{acres}}\right)\left(\frac{1609\,\text{m}}{\text{mi}}\right)^2$$

$$= \boxed{4050\,\text{m}^2}$$

**32** ••

**Picture the Problem** The volume of a right circular cylinder is the area of its base multiplied by its height. Let $d$ represent the diameter and $h$ the height of the right circular cylinder; use conversion factors to express the volume $V$ in the given units.

(*a*) Express the volume of the cylinder:

$$V = \tfrac{1}{4}\pi d^2 h$$

Substitute numerical values and evaluate $V$:

$$V = \tfrac{1}{4}\pi(6.8\,\text{in})^2(2\,\text{ft})$$

$$= \tfrac{1}{4}\pi(6.8\,\text{in})^2(2\,\text{ft})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^2$$

$$= \boxed{0.504\,\text{ft}^3}$$

(*b*) Use the fact that 1 m = 3.281 ft to convert the volume in cubic feet into cubic meters:

$$V = (0.504\,\text{ft}^3)\left(\frac{1\,\text{m}}{3.281\,\text{ft}}\right)^3$$

$$= \boxed{0.0143\,\text{m}^3}$$

(*c*) Because 1 L = $10^{-3}$ m$^3$:

$$V = (0.0143\,\text{m}^3)\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right) = \boxed{14.3\,\text{L}}$$

***33 ••**

**Picture the Problem** We can treat the SI units as though they are algebraic quantities to simplify each of these combinations of physical quantities and constants.

(*a*) Express and simplify the units of $v^2/x$:

$$\frac{(\mathrm{m/s})^2}{\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{m}\cdot\mathrm{s}^2} = \boxed{\frac{\mathrm{m}}{\mathrm{s}^2}}$$

(*b*) Express and simplify the units of $\sqrt{x/a}$:

$$\sqrt{\frac{\mathrm{m}}{\mathrm{m/s}^2}} = \sqrt{\mathrm{s}^2} = \boxed{\mathrm{s}}$$

(*c*) Noting that the constant factor $\frac{1}{2}$ has no units, express and simplify the units of $\frac{1}{2}at^2$:

$$\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)(\mathrm{s})^2 = \left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)(\mathrm{s}^2) = \boxed{\mathrm{m}}$$

## Dimensions of Physical Quantities

**34 •**

**Picture the Problem** We can use the facts that each term in an equation must have the same dimensions and that the arguments of a trigonometric or exponential function must be dimensionless to determine the dimensions of the constants.

(*a*)

$$\begin{array}{ccccccc} x & = & C_1 & + & C_2 & t \\ L & & \boxed{L} & & \boxed{\frac{L}{T}} & T \end{array}$$

(*b*)

$$\begin{array}{ccccc} x & = & \frac{1}{2} & C_1 & t^2 \\ L & & & \boxed{\frac{L}{T^2}} & T^2 \end{array}$$

(*c*)

$$\begin{array}{cccccc} v^2 & = & 2 & C_1 & x \\ \frac{L^2}{T^2} & & & \boxed{\frac{L}{T^2}} & L \end{array}$$

(*d*)

$$\begin{array}{cccccc} x & = & C_1 & \cos & C_2 & t \\ L & & \boxed{L} & & \boxed{\frac{1}{T}} & T \end{array}$$

(*e*)

$$\begin{array}{cccccc} v & = & C_1 & \exp( & -C_2 & t) \\ \frac{L}{T} & & \boxed{\frac{L}{T}} & & \boxed{\frac{1}{T}} & T \end{array}$$

**35 ••**

**Picture the Problem** Because the exponent of the exponential function must be dimension the dimension of $\lambda$ must be $\boxed{T^{-1}.}$

***36** ••

**Picture the Problem** We can solve Newton's law of gravitation for $G$ and substitute the dimensions of the variables. Treating them as algebraic quantities will allow us to express the dimensions in their simplest form. Finally, we can substitute the SI units for the dimensions to find the units of $G$.

Solve Newton's law of gravitation for $G$ to obtain:

$$G = \frac{Fr^2}{m_1 m_2}$$

Substitute the dimensions of the variables:

$$G = \frac{\frac{ML}{T^2} \times L^2}{M^2} = \boxed{\frac{L^3}{MT^2}}$$

Use the SI units for $L$, $M$, and $T$:

$$\text{Units of } G \text{ are } \boxed{\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}}$$

**37** ••

**Picture the Problem** Let $m$ represent the mass of the object, $v$ its speed, and $r$ the radius of the circle in which it moves. We can express the force as the product of $m$, $v$, and $r$ (each raised to a power) and then use the dimensions of force $F$, mass $m$, speed $v$, and radius $r$ to obtain three equations in the assumed powers. Solving these equations simultaneously will give us the dependence of $F$ on $m$, $v$, and $r$.

Express the force in terms of powers of the variables:

$$F = m^a v^b r^c$$

Substitute the dimensions of the physical quantities:

$$MLT^{-2} = M^a \left(\frac{L}{T}\right)^b L^c$$

Simplify to obtain:

$$MLT^{-2} = M^a L^{b+c} T^{-b}$$

Equate the exponents to obtain:

$a = 1$,
$b + c = 1$, and
$-b = -2$

Solve this system of equations to obtain:

$a = 1$, $b = 2$, and $c = -1$

Substitute in equation (1):

$$F = mv^2 r^{-1} = \boxed{m\frac{v^2}{r}}$$

**38** ••

**Picture the Problem** We note from Table 1-2 that the dimensions of power are $ML^2/T^3$. The dimensions of mass, acceleration, and speed are M, $L/T^2$, and L/T respectively.

Express the dimensions of *mav*:

$$[mav] = M \times \frac{L}{T^2} \times \frac{L}{T} = \frac{ML^2}{T^3}$$

From Table 1-2:

$$[P] = \frac{ML^2}{T^3}$$

Comparing these results, we see that the product of mass, acceleration, and speed has the dimensions of power.

**39** ••

**Picture the Problem** The dimensions of mass and velocity are M and L/T, respectively. We note from Table 1-2 that the dimensions of force are $ML/T^2$.

Express the dimensions of momentum:

$$[mv] = M \times \frac{L}{T} = \frac{ML}{T}$$

From Table 1-2:

$$[F] = \frac{ML}{T^2}$$

Express the dimensions of force multiplied by time:

$$[Ft] = \frac{ML}{T^2} \times T = \frac{ML}{T}$$

Comparing these results, we see that momentum has the dimensions of force multiplied by time.

**40** ••

**Picture the Problem** Let *X* represent the physical quantity of interest. Then we can express the dimensional relationship between *F, X,* and *P* and solve this relationship for the dimensions of *X*.

Express the relationship of *X* to force and power dimensionally:

$$[F][X] = [P]$$

Solve for $[X]$:

$$[X] = \frac{[P]}{[F]}$$

Substitute the dimensions of force and power and simplify to obtain:

$$[X]=\frac{\frac{ML^2}{T^3}}{\frac{ML}{T^2}}=\frac{L}{T}$$

Because the dimensions of velocity are $L/T$, we can conclude that:

$$\boxed{[P]=[F][v]}$$

**Remarks: While it is true that *P* = *Fv*, dimensional analysis does not reveal the presence of dimensionless constants. For example, if $P=\pi Fv$, the analysis shown above would fail to establish the factor of $\pi$.**

***41 ••**

**Picture the Problem** We can find the dimensions of *C* by solving the drag force equation for *C* and substituting the dimensions of force, area, and velocity.

Solve the drag force equation for the constant *C*:

$$C=\frac{F_{\text{air}}}{Av^2}$$

Express this equation dimensionally:

$$[C]=\frac{[F_{\text{air}}]}{[A][v]^2}$$

Substitute the dimensions of force, area, and velocity and simplify to obtain:

$$[C]=\frac{\frac{ML}{T^2}}{L^2\left(\frac{L}{T}\right)^2}=\boxed{\frac{M}{L^3}}$$

**42 ••**

**Picture the Problem** We can express the period of a planet as the product of these factors (each raised to a power) and then perform dimensional analysis to determine the values of the exponents.

Express the period *T* of a planet as the product of $r^a$, $G^b$, and $M_S^c$:

$$T=Cr^aG^bM_S^c \qquad (1)$$

where *C* is a dimensionless constant.

Solve the law of gravitation for the constant *G*:

$$G=\frac{Fr^2}{m_1m_2}$$

Express this equation dimensionally:

$$[G]=\frac{[F][r]^2}{[m_1][m_2]}$$

Substitute the dimensions of $F$, $r$, and $m$:

$$[G] = \frac{\frac{ML}{T^2}\times(L)^2}{M\times M} = \frac{L^3}{MT^2}$$

Noting that the dimension of time is represented by the same letter as is the period of a planet, substitute the dimensions in equation (1) to obtain:

$$T = (L)^a\left(\frac{L^3}{MT^2}\right)^b(M)^c$$

Introduce the product of $M^0$ and $L^0$ in the left hand side of the equation and simplify to obtain:

$$M^0L^0T^1 = M^{c-b}L^{a+3b}T^{-2b}$$

Equate the exponents on the two sides of the equation to obtain:

$0 = c - b$,
$0 = a + 3b$, and
$1 = -2b$

Solve these equations simultaneously to obtain:

$$a = \tfrac{3}{2}, b = -\tfrac{1}{2}, \text{and } c = -\tfrac{1}{2}$$

Substitute in equation (1):

$$T = Cr^{3/2}G^{-1/2}M_S^{-1/2} = \boxed{\frac{C}{\sqrt{GM_S}}r^{3/2}}$$

## Scientific Notation and Significant Figures

***43 •**

**Picture the Problem** We can use the rules governing scientific notation to express each of these numbers as a decimal number.

(*a*) $3\times10^4 = \boxed{30{,}000}$

(*b*) $6.2\times10^{-3} = \boxed{0.0062}$

(*c*) $4\times10^{-6} = \boxed{0.000004}$

(*d*) $2.17\times10^5 = \boxed{217{,}000}$

**44 •**

**Picture the Problem** We can use the rules governing scientific notation to express each of these measurements in scientific notation.

(*a*) $3.1\text{GW} = \boxed{3.1\times10^9\text{ W}}$

(*c*) $2.3\text{fs} = \boxed{2.3\times10^{-15}\text{ s}}$

(*b*) $10\,\text{pm} = 10\times10^{-12}\text{ m} = \boxed{10^{-11}\text{ m}}$ (*d*) $4\,\mu\text{s} = \boxed{4\times10^{-6}\text{ s}}$

**45** •

**Picture the Problem** Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) The number of significant figures in each factor is three; therefore the result has three significant figures:

$$(1.14)(9.99\times10^{4}) = \boxed{1.14\times10^{5}}$$

(*b*) Express both terms with the same power of 10. Because the first measurement has only two digits after the decimal point, the result can have only two digits after the decimal point:

$$(2.78\times10^{-8})-(5.31\times10^{-9}) = (2.78-0.531)\times10^{-8} = \boxed{2.25\times10^{-8}}$$

(*c*) We'll assume that 12 is exact. Hence, the answer will have three significant figures:

$$\frac{12\pi}{4.56\times10^{-3}} = \boxed{8.27\times10^{3}}$$

(*d*) Proceed as in (*b*):

$$27.6+(5.99\times10^{2}) = 27.6+599 = 627 = \boxed{6.27\times10^{2}}$$

**46** •

**Picture the Problem** Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Note that both factors have four significant figures.

$$(200.9)(569.3) = \boxed{1.144\times10^{5}}$$

(*b*) Express the first factor in scientific notation and note that both factors have three significant figures.

$$(0.000000513)(62.3\times10^{7}) = (5.13\times10^{-7})(62.3\times10^{7}) = \boxed{3.20\times10^{2}}$$

(*c*) Express both terms in scientific notation and note that the second has only three significant figures. Hence the result will have only three significant figures.

$$28401+\left(5.78\times10^{4}\right) = \left(2.841\times10^{4}\right)+\left(5.78\times10^{4}\right) = \left(2.841+5.78\right)\times10^{4} = \boxed{8.62\times10^{4}}$$

(*d*) Because the divisor has three significant figures, the result will have three significant figures.

$$\frac{63.25}{4.17\times10^{-3}} = \boxed{1.52\times10^{4}}$$

***47** •

**Picture the Problem** Let *N* represent the required number of membranes and express *N* in terms of the thickness of each cell membrane.

Express *N* in terms of the thickness of a single membrane:

$$N = \frac{1\,\text{in}}{7\,\text{nm}}$$

Convert the units into SI units and simplify to obtain:

$$N = \frac{1\,\text{in}}{7\,\text{nm}}\times\frac{2.54\,\text{cm}}{\text{in}}\times\frac{1\,\text{m}}{100\,\text{cm}}\times\frac{1\,\text{nm}}{10^{-9}\,\text{m}} = \boxed{4\times10^{6}}$$

**48** •

**Picture the Problem** Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Both factors and the result have three significant figures:

$$\left(2.00\times10^{4}\right)\left(6.10\times10^{-2}\right) = \boxed{1.22\times10^{3}}$$

(*b*) Because the second factor has three significant figures, the result will have three significant figures:

$$\left(3.141592\right)\left(4.00\times10^{5}\right) = \boxed{1.26\times10^{6}}$$

(*c*) Both factors and the result have three significant figures:

$$\frac{2.32\times10^{3}}{1.16\times10^{8}} = \boxed{2.00\times10^{-5}}$$

(*d*) Write both terms using the same power of 10. Note that the result will have only three significant figures:

$$\left(5.14\times10^{3}\right)+\left(2.78\times10^{2}\right) = \left(5.14\times10^{3}\right)+\left(0.278\times10^{3}\right) = \left(5.14+0.278\right)\times10^{3} = \boxed{5.42\times10^{3}}$$

(*e*) Follow the same procedure used in (*d*):

$$\left(1.99\times10^{2}\right)+\left(9.99\times10^{-5}\right)$$
$$=\left(1.99\times10^{2}\right)+\left(0.000000999\times10^{2}\right)$$
$$=\boxed{1.99\times10^{2}}$$

***49** •

**Picture the Problem** Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) The second factor and the result have three significant figures:

$$3.141592654\times(23.2)^2=\boxed{1.69\times10^{3}}$$

(*b*) We'll assume that 2 is exact. Therefore, the result will have two significant figures:

$$2\times3.141592654\times0.76=\boxed{4.8}$$

(*c*) We'll assume that 4/3 is exact. Therefore the result will have two significant figures:

$$\frac{4}{3}\pi\times(1.1)^3=\boxed{5.6}$$

(*d*) Because 2.0 has two significant figures, the result has two significant figures:

$$\frac{(2.0)^5}{3.141592654}=\boxed{10}$$

## General Problems

**50** •

**Picture the Problem** We can use the conversion factor 1 mi = 1.61 km to convert 100 km/h into mi/h.

Multiply 100 km/h by 1 mi/1.61 km to obtain:

$$100\frac{\text{km}}{\text{h}}=100\frac{\text{km}}{\text{h}}\times\frac{1\text{mi}}{1.61\text{km}}$$
$$=\boxed{62.1\text{mi/h}}$$

***51** •

**Picture the Problem** We can use a series of conversion factors to convert 1 billion seconds into years.

Multiply 1 billion seconds by the appropriate conversion factors to convert into years: